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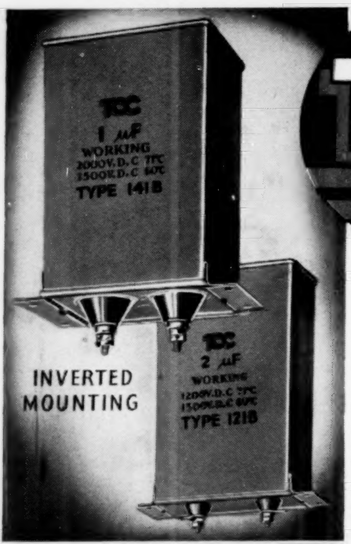

TCC
1 μF
WORKING
2000V.D.C 71°C
1500V.D.C 60°C
TYPE 141B
INVERTED MOUNTING
TCC
2 μF
WORKING
1200V.D.C 71°C
1500V.D.C 60°C
TYPE 121B

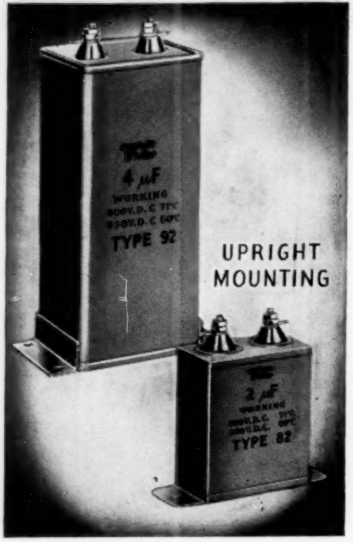
TROPICAL PAPER Capacitors

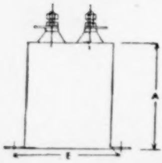
IN RECTANGULAR METAL CANS

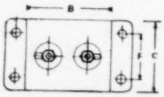
TYPE	CAPACITANCE μF	DIMENSIONS in INCHES					LIST PRICE EACH	
		A	B	C	E	F	s.	d.
TYPE 62 350 v. D.C. working (at 60° C)	*0.25	2⅜	1¾	¾	2⅛	—	5	3
	*0.5	2⅜	1¾	¾	2⅛	—	5	6
	*1.0	2⅜	1¾	¾	2⅛	—	6	6
	*2.0	2⅜	1¾	1	2⅛	—	7	9
	*4.0	2⅜	1¾	1⅜	2⅛	—	11	6
	8.0	4.11/16	1⅝	2⅜	2	1⅞	20	9
TYPE 82 500 v. D.C. working (at 60° C)	*0.25	2	2	¾	2⅜	—	5	6
	*0.5	2⅜	2	¾	2⅜	—	5	9
	*1.0	2⅜	2	¾	2⅜	—	7	0
	2.0	2⅜	2⅛	1¼	2½	—	8	9
	4.0	2.9/16	2⅛	2⅜	2½	1⅝	15	0
	8.0	4.11/16	1⅝	3⅛	2	2⅜	25	3
TYPE 92 750 v. D.C. working (at 60° C)	*0.25	2⅜	2	¾	2⅜	—	6	0
	*0.5	2⅜	2	¾	2⅜	—	6	6
	*1.0	2⅜	2	1	2⅜	—	7	6
	2.0	2.9/16	2⅛	2⅛	2½	1⅜	12	9
	4.0	4.11/16	2⅛	2⅛	2½	1⅜	18	6
	8.0	4.11/16	2⅛	4⅛	2½	3⅜	31	0
TYPE 111 1,000 v. D.C. working (at 60° C)	*0.25	2⅜	2	¾	2½	—	6	9
	*0.5	2⅜	2	¾	2½	—	7	9
	1.0	2.9/16	2⅛	1⅜	2½	—	11	0
	2.0	4.15/16	2⅛	1⅜	2½	—	15	9
	4.0	4.15/16	4⅛	1⅜	4½	—	22	6
	8.0	4.15/16	4⅛	2⅜	4½	1⅝	40	3
TYPE 121B 1,500 v. D.C. working (at 60° C)	*0.1	2⅜	2	¾	2½	—	7	0
	0.5	2.9/16	2⅝	1⅜	3	—	10	0
	1.0	4.15/16	2¼	1⅛	2⅝	—	13	9
	2.0	4.15/16	3⅛	1⅜	3½	—	19	0
	4.0	4.15/16	3⅛	2⅝	3½	1⅞	31	0
	8.0	4.15/16	5⅛	3⅛	5½	2⅜	59	9
TYPE 131 2,000 v. D.C. working (at 60° C)	0.1	2.9/16	2¼	1⅛	2⅝	—	8	0
	0.5	4.15/16	2¼	1⅛	2⅝	—	11	9
	1.0	4.15/16	2⅝	1⅝	3	—	15	6
	2.0	4.15/16	2⅝	2⅝	3	1⅞	24	9
	4.0	4.15/16	4⅞	2⅝	5¼	1⅞	41	6
	8.0	4.15/16	4⅞	5⅛	5¼	4⅜	81	9

NOTE: If inverted mounting (through chassis) style required, add suffix "IM," when ordering.

** Denotes plain soldered seams (others rolled seams).*


TCC
4 μF
WORKING
800V.D.C 71°C
750V.D.C 60°C
TYPE 92
UPRIGHT MOUNTING
TCC
2 μF
WORKING
500V.D.C. 71°C
500V.D.C. 60°C
TYPE 82





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Vol. XXV JUNE 1948 No. 297

Kirchhoff's First and Second Laws

UNTIL quite recently we were not aware that there was any doubt as to the meaning of Kirchhoff's first and second laws, but on two occasions lately we have come across statements in which the normal order was reversed and the second law referred to as the first. On the first occasion, in a recently published book on circuit theory, we assumed that it was just a slip, but in an article on network theory by W. H. Ingram in the *American Journal of Mathematics and Physics* the author states that he has deliberately chosen " to follow the numbering finally chosen by Kirchhoff," although, as he says, " Many "—he might have said practically all—" textbooks follow the numerical designation of his laws given in his original communications." It is true that Kirchhoff published two papers* on the subject and that in the second paper the order of the two laws was reversed.

In his 1845 paper he said, " (1) If the wires 1, 2, . . . μ meet at a point, $I_1 + I_2 + \ldots + I_\mu = 0$. (2) If the wires 1, 2, . . . ν form a closed figure, $I_1\omega_1 + I_2\omega_2 + \ldots + I_\nu \omega_\nu$ = the sum of all the electromotive forces in the path 1, 2, ... ν, where $\omega_1, \omega_2 \ldots$ are the resistances of the wires, and $I_1, I_2, \ldots$ the currents, all in one direction being regarded as positive." In his 1847 paper he uses very similar words but writes

I. $\omega_{k1}I_{k1}+\omega_{k2}I_{k2}+\ldots=E_{k1}+E_{k2}+\ldots$

II. $I_{\lambda 1}+I_{\lambda 2}+\ldots.=0$.

For nearly a hundred years the 1845 designation has been universally adopted, perhaps not entirely because it was Kirchhoff's original order, but because of the obvious simplicity of the first law, involving, as it does, nothing but currents. It is misleading to refer to the 1847 order as that " finally *chosen* by Kirchhoff." He probably regarded the order as immaterial. He would certainly have been very surprised had he known that after a hundred years his reversal of the original order was to be made the excuse for muddling up the universally recognized designation.

We have searched along the library shelves through a number of textbooks in English, French and German to see if we could find any support for this queer procedure, but we have found none except in a book recently published in America. There may, of course, be others among post-war American publications, but it can be confidently stated that the vast majority of textbooks published during the last hundred years, including all the classics, have used the original designation.

If one is dissatisfied with this it would surely be better for him to give up the use of first and second, and refer to them as the star or junction law and the mesh or circuit law. Another question that is sometimes raised is whether they should be called laws. In various German publications one finds them referred to as Gesetze, Regeln, Sätze, Bedingungen, and Gleichungen, which may be translated as laws, rules, propositions, conditions, and equations. It may be argued that they are not laws but merely rules based on Ohm's law. Here again, however, the term " Kirchhoff's Laws " has become almost standardized and certainly has the virtue of brevity.

G. W. O. H.

**Annalen der Physik und Chemie,* 1845, Vol. 64, p. 513 and 1847, Vol. 72, p. 497.

H-TYPE ADCOCK DIRECTION FINDERS

Design Principles for 3-30 Mc/s

By W. Ross, M.A., A.M.I.E.E. and R. E. Burgess, B.Sc.

(Communication from the National Physical Laboratory)

SUMMARY.—The paper sets out in general terms the factors which are of importance in the design of H-Adcock direction finders for high frequencies (3-30 Mc/s). It is based on the experience gained over several years in the design of practical H-type direction finders as well as on the results of experiments specifically designed to investigate certain problems of design.

The main conclusions are :—

(*a*) For maximum sensitivity it is important to maintain as high a ratio of aerial capacitance to total parallel aerial-circuit capacitance as possible and to keep the effective length of the dipoles as large as possible. Both of these requirements are met by end-loading of the dipoles.

(*b*) The design of the coupling circuits between aerials and the first valve of the receiver should be such as to ensure (1) a high coefficient of coupling in all transformers or goniometers ; (2) as few intermediate circuits as possible between aerials and receiver ; (3) the primary inductance of the aerial transformers (or goniometer field-coils) should resonate with the total aerial-circuit capacitance just outside the high-frequency end of each frequency band ; and (4) the total working-frequency range should be covered in a series of sub-ranges each of the order of 1.5 to 1 in frequency.

(*c*) Aerial-circuit balancing becomes extremely critical in the neighbourhood of the fundamental resonant frequency of the central column formed by the down leads and/or supporting column.

(*d*) In small rotating-aerial systems the presence of an operator may, when for any reason the bearings are flat, cause serious bearing errors, even when he sits at what appears to be the directional minimum.

(*e*) The image in the ground of the horizontal conductor formed by the feeder lines (including any associated screening tube) gives rise to a polarization error inherent in H-aerial systems but only of importance when the aerials are at a height above ground small compared with their spacing.

(*f*) Another known source of polarization error which occurs particularly with portable rotating-aerial direction finders is that due to the presence of the receiver box between, or in proximity to, the lower halves of the dipoles.

(*g*) There are other, as yet unexplained, sources of polarization error which require further study.

1. General Introduction

THE H-Adcock direction finder has certain advantages which make it the most suitable type of Adcock system to employ in certain circumstances. It has the greatest inherent symmetry and is the least likely to be affected by the nature of the ground on which it is erected. Moreover, both these qualities are enhanced, the smaller the aerial system can be made, since the height of the aerial system above the ground relative to the length of the dipole is thereby easily made large without the necessity of raising the centre of the system to an inconvenient height.

The present paper discusses various aspects of design which arose in the development of elevated two-aerial and four-aerial systems for frequencies in the range 3–30 Mc/s approximately. In some types the operating position was required to be at an appreciable distance below the aerials, so that an important feature of the systems discussed is the existence of a long vertical conducting column at the centre of the aerials. The two-aerial rotating systems may be represented as in Fig. 1 where (a) shows the system with a tuned primary circuit and (b) the system with an untuned primary.

Four-aerial systems which utilize a goniometer may be classified into the two types shown in Fig. 2, viz.,

(a) *The directly-coupled system*

The goniometer and the r.f. stages of the receiver are situated at the centre of the aerial system and are controlled remotely from the ground ; the i.f. signal is fed down

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MS accepted by the Editor, January 1947.

a cable to the i.f. and l.f. stages of the receiver. This type of circuit enables the highest possible sensitivity to be obtained from a given aerial system.

(b) *The line-coupled system*

The Adcock aerial pairs are coupled through screened transformers to a pair of transmission lines which carry the signal-frequency currents down to the field-coils of the goniometer. This system has a lower sensitivity than the first type on account of the loss in the transmission line and of the difficulty of accurate matching over a wide frequency band.

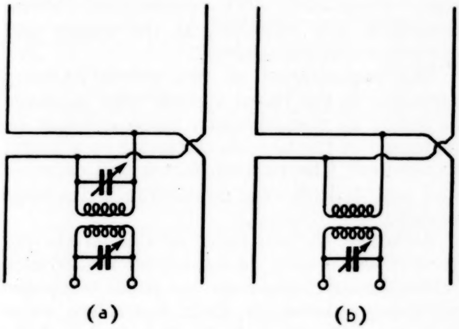

Fig. 1. *Basic circuits of two-aerial H-Adcock system: (a) tuned primary; (b) untuned primary.*

The design of these systems will be discussed under the separate headings of sensitivity, directional accuracy and polarization error.

2. Sensitivity

The sensitivity of a direction finder is determined by the smallest field intensity of signal with which bearings can be determined to a specified degree of accuracy. Background noise is implicit in considerations of sensitivity† and for this reason "pick-up factor," introduced in an earlier paper[1], is not wholly satisfactory although many of the principles of design for maximum pick-up factor also apply to sensitivity.

The background noise considered is that arising in the circuits (thermal noise) and in the valves (shot noise). Noise from external sources which is received on the aerials will in practice also be present, but this can only be treated in the most general qualitative way. It is sufficient to say that the presence of external noise does not displace the coupling conditions for optimum signal/noise ratio, but only makes them less critical. The ultimate limitation to sensitivity is the thermal noise generated in the circuit elements between the aerials and the first valve; at frequencies above about 10 Mc/s the shot noise in the first valve becomes significant and it is then necessary to minimize the valve noise.

In considering the design of an H-Adcock system for maximum sensitivity it is convenient to consider the components of the system separately; viz., the aerials and feeders, the coupling circuit, and the input circuit of the receiver. In discussing the aerial circuit it must be remembered that considerations of balance, whether of an aerial pair, or of two pairs relative to each other in a four-aerial system, may override considerations of sensitivity.

2.1.—*Aerial system*

In a four-aerial system (Fig. 2) independent tuning of the two aerial pairs is made impracticable by the difficulty of maintaining equality of amplitude and phase in the two channels, and for this reason the system must be untuned as far as the goniometer search coil. Nevertheless it is convenient to consider a four-aerial system in terms of the "equivalent" two-aerial system which has the same coupling elements, with a transformer having parameters equal to those of the goniometer in its position of maximum coupling.

Considering the two-aerial system, it is found that direct connection of the aerials to the receiver is undesirable because of the high degree of admittance balance to earth required.

The systems at present under consideration, shown in Figs. 1 and 2, may be discussed generally in relation to the influence of the parameters of the aerial system on the sensitivity. In this paper we shall consider only high-frequency systems for which the aerial dimensions are small compared with the wavelength. With this limitation, to obtain maximum sensitivity for an aerial system of given dimensions and working on a given wavelength the objects of design must be to make :—

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† "Sensitivity" is used in accordance with the British Standard "Glossary of Terms used in Telecommunication," B.S.204 (1943).

(i) the ratio $\dfrac{\text{effective length}}{\text{true length}}$ as large as possible ;

(ii) the aerial capacitance as large as possible ;

(iii) the feeder capacitance and fixed capacitance associated therewith as small as possible ;

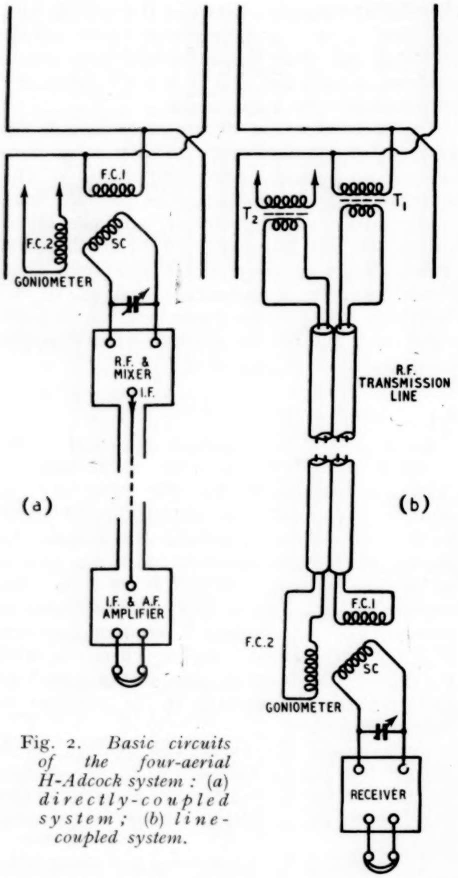

Fig. 2. *Basic circuits of the four-aerial H-Adcock system : (a) directly-coupled system ; (b) line-coupled system.*

(iv) the primary tuning capacitance in the tuned system as small as possible ;

(v) the magnification factor as large as possible.

The obvious means of increasing the aerial capacitance without altering the actual height of the aerials is to increase their superficial area. If this is done by loading the extremities of the aerial with discs or spheres the effective height is thereby increased at the same time, and may in fact be made to approach closely to the true height. For example, by fixing light metallic spheres 15 cm in diameter to the extremities of dipoles of approximately one metre length the aerial capacitance is doubled and the effective height increased by about 50 per cent.

The third point is dealt with by spacing the feeders as far apart as possible, by using the smallest possible wire consistent with the requirements of (v), and by reducing to the lowest possible limit the self-capacitance of the primary inductance and all stray capacitance associated with connecting wires, terminals, and switches, at the centre and extremities of the feeders.

The capacitance of the primary-tuning capacitor in the tuned system must be made as small as possible but, since it must be increased in the process of tuning to a longer wavelength, the pick-up factor will decrease and may fall off very considerably if a large waveband is desired.

Although the reactance of the aerials and feeders will usually be capacitive in a system whose linear dimensions are small compared with the wavelength, their equivalent series inductance may result in a resonance of the aerial circuit at the higher frequencies in the band. For example, in an experimental system 2 m square, such a resonance occurred at about 18 Mc/s. It is found that the reactance of the aerial and feeder circuit can be accurately represented by a series combination of capacitance and inductance for frequencies up to and some way beyond this resonance.

The resistance of the aerial circuit is mainly ohmic in the relatively small systems considered, for the radiation resistance of the Adcock aerial pair is low. For example an aerial system having dipoles of effective length 1.5 metres and spacing of 2 metres has a radiation resistance at 10 Mc/s of only 0.036 ohm. In fact, the fundamental reason for the low sensitivity of the small Adcock system is the preponderance of ohmic resistance over radiation resistance.

2.2 *Aerial coupling circuit*

Since for reasons mentioned elsewhere, the untuned primary circuit is preferable to the tuned primary circuit in the two-aerial system, and it is essential in the four-aerial system, this type is now considered in detail.

The aerial circuit is normally capacitive and the condition for maximum sensitivity[2] is that the coefficient of coupling k of the aerial transformer, or of the goniometer in its maximum position, shall be as large as possible, and in practice this implies a value of k of about 0.5. Furthermore, the inductance of the primary (or field-coil) should be such that it resonates with the aerial capacitance at a frequency a little outside one end of the band. How near to the band this resonance should lie is determined by the impedance reflected from the primary into the secondary. Fig. 3 shows the variation of the effective reactance X' and magnification Q' of the secondary in the region of the primary resonance frequency f_1.

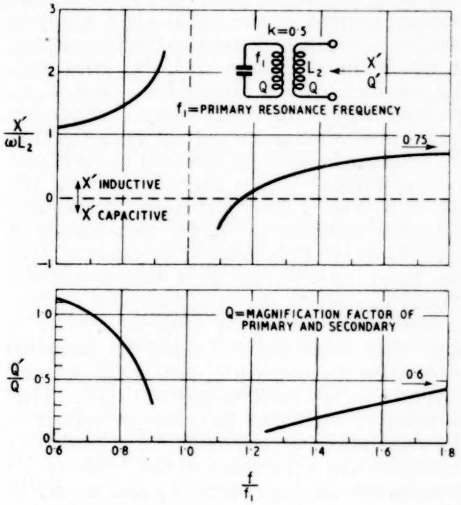

Fig. 3 (above). *Behaviour of the effective reactance X' and the effective magnification Q' of the secondary in the region of the primary resonance frequency.*

In the directly-coupled system it is required to tune the search coil of the goniometer to form the first circuit of the receiver. For this reason the primary resonance must not be too near to the edge of the band, for otherwise excessive reactance and resistance is reflected from the field-coil circuit, making tuning difficult and reducing the dynamic impedance of the search coil, so increasing the effect of valve noise. It is found in practice that the primary resonance should not be closer to the edges of the band than about 0.7 of the lowest operating frequency $f_{min}\left(\frac{f_{min}}{f_1} = 1.4\right)$ or about 1.25 times the the highest frequency $f_{max}\left(\frac{f_{max}}{f_1} = 0.8\right)$.

In Fig. 4 the ratio of signal to circuit noise over a frequency band of ratio 1.6 is shown for an input transformer (or goniometer) having $k = 0.5$ and equal magnification Q in the two windings which is independent of frequency. The signal e.m.f. from the Adcock pair is proportional to the frequency and the resistance of the primary circuit is assumed to occur wholly in the inductance. When the primary resonance frequency f_1 is 0.7 of the lowest working frequency f_{min}, the sensitivity is practically uniform over the band, but when it is 1.25 times the highest working frequency f_{max} the sensitivity rises steadily with increasing frequency. In the first case the increase of signal with frequency is offset by the loss due to receding from the primary resonance, while in the latter the signal increase is accompanied by an approach to the resonance. Thus the more uniform sensitivity is obtained when the primary resonance is outside the lower edge of the band, but the sensitivity is on the average at a distinctly lower level than when

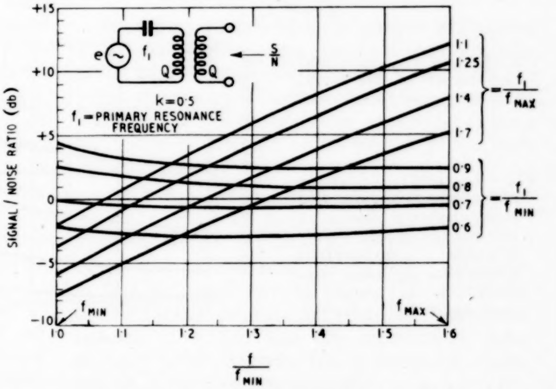

Fig. 4 (right). *Relative signal/noise ratio at the secondary over a frequency band of ratio 1.6 for various primary resonance frequencies ; f/f_{min} = ratio of operating frequency to the minimum frequency of the band.*

the resonance is outside the upper edge of the band.

In the line-coupled system, analysis of the overall sensitivity is considerably complicated by the variability of the impedance presented by the transmission line which is connected at its top end to the substantially reactive impedance of the aerial transformer. Since the signal/noise ratio should be as large as possible at the terminals of the secondary, it appears that there will be some advantage in operating with the primary resonance quite close to the edge of the working band. Furthermore, by reflecting as much resistance as possible from the primary to the secondary, the loss in the transmission line will be reduced.

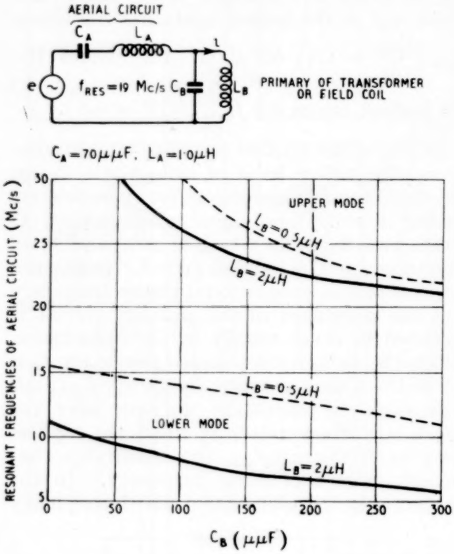

Fig. 5 (above). *Resonant frequencies of the aerial circuit for a typical case.*

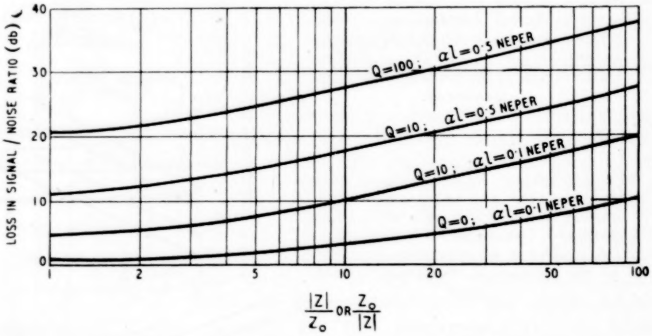

Fig. 6 (right). *Loss in signal/noise ratio due to a transmission line of attenuation αl and purely resistive characteristic impedance Z_0 when connected to a source of impedance $|Z|/\underline{\pm \tan^{-1} Q}$.*

It was mentioned in the previous section that the impedance of the aerial and feeder circuit connected to the primary or field-coil may resonate within the overall band of the instrument. The aerial circuit (Fig. 5) may be represented to a first approximation by a capacitance C_A and inductance L_A in series, while the primary coil L_B will have associated parallel capacitance C_B which may consist of self-capacitance and any added loading capacitance.

It may be shown that the ratio of the primary current to the aerial e.m.f. exhibits two resonances which are also the frequencies at which the reactance of L_B in series with the parallel combination of C_B and $C_A L_A$ becomes zero. It is therefore undesirable that these resonances should occur within the working band or be too near to its edges. Taking typical values for C_A and L_A the curves in Fig. 5 show the resonant frequencies as a function of C_B for two values of L_B. Thus, if it is desired to operate over the band 12.5 to 20 Mc/s the resonances should lie outside the limits 8.8 and 25 Mc/s in the directly-coupled system, and it is seen that these limits cannot be complied with when $L_B = 0.5\ \mu H$, but if $L_B = 2\ \mu H$, C_B must lie between 80 and 120 $\mu\mu F$. Thus in order to avoid the too close proximity of the resonances to the band, it is necessary to choose the inductance of the field-coil (or transformer primary) suitably and it may be necessary to use additional capacitive loading although, in practice, this is always detrimental to sensitivity.

In the line-coupled system the secondary of the transformer and the field-coil of the goniometer are connected to the ends of the transmission line. Considerations of sensitivity[2] show that these windings should be so proportioned that their reactance is approximately equal to the characteristic impedance of the transmission line.

Fig 6 shows the reduction in signal/noise ratio at the end of a line (of attenuation αl and resistive characteristic impedance Z_0) below the value at the source to which it is connected when the source has an impedance of modulus $|Z|$ and ratio Q of reactance to resistance. In the case under consideration the impedance of the source is the effective impedance looking into the secondary of the aerial transformer. The curves clearly demonstrate the need for using a line of low attenuation and of reflecting as much resistance as possible from the primary to the secondary by using close coupling.

2.3. *Input circuit of the receiver*

For maximum sensitivity the search coil of the goniometer should be tuned and form the first grid circuit of the receiver. Appreciable loss of sensitivity will result if an intermediate coupling circuit is used.

The inductance of the search coil or secondary is determined by tuning considerations and the coefficient of coupling with the field-coils should be as large as possible. The magnification of the windings should be high and the self-capacitance low in order that the tuning capacitance for a given frequency band shall be as small as possible; this enables a high dynamic impedance to be obtained and thus the contribution of the first valve to the background noise is minimized.

The dynamic impedance will be reduced on account of the resistance (and reactance, if negative) reflected from the field-coil or primary circuits. In the directly-coupled aerial system the dynamic impedance is seen from Fig. 3 to be appreciably greater when the primary resonance is above the band than when below it, and this is a further argument for using the former condition.

When the search-coil tuning capacitor is ganged with other tuning controls and no separate trimmer is provided, inaccuracies of ganging will result in a reduction of the equivalent parallel resistance D of the circuit below the dynamic impedance D_0. If valve noise were absent, misganging would not affect the signal/noise ratio for the signal and thermal noise are reduced in the same ratio. The presence of valve noise, however makes it desirable to provide a separate trimmer capacitor for the search-coil tuning if maximum sensitivity is to be achieved.

An important limitation to the accuracy of ganging of the tuned search coil may arise from variability of the reactance reflected from the field-coil circuits, when the search coil is rotated. Thus if Z_1 and Z_2 are the impedances of the field-coil circuits and M_1 and M_2 are their mutual inductances to the search coil, the impedance reflected into the search coil is

$$Z' = \frac{\omega^2 M_1^2}{Z_1} + \frac{\omega^2 M_2^2}{Z_2}$$

Ideally Z_1 and Z_2 should be equal, and M_1 and M_2 should have the form $M \sin \theta$ and $M \cos \theta$, in which case Z' is constant. Inequalities in the impedances of the field-coil circuits or inaccuracies in the coupling law of the goniometer will result in the reflected impedance being dependent on the angular setting of the search coil. Misganging and variable damping of the search coil are thereby produced and the effect on sensitivity may not be negligible; the effect can usually be observed, if present, as a variation of circuit noise as the search coil is rotated. A secondary effect is the appearance of spurious minima in the signal, although in practice these are so flat as not to cause confusion. Displacement of the normal signal minimum will occur when this is flat but the effect is not of serious practical importance.

3. Directional Accuracy

In a four-aerial Adcock system unbalance in an aerial pair gives rise to a semi-circular error while amplitude inequality between the pairs gives a quadrantal error. The resultant error for a wave arriving at angle θ with the plane of the NS aerial pair is given very approximately by

$$\epsilon = \epsilon_1 \cos \theta + \epsilon_2 \sin \theta + \epsilon_0 \sin 2\theta$$

where ϵ_1 and ϵ_2 are the errors for waves arriving from N and E respectively and ϵ_0 is the quadrantal component of error. A further error due to the aerial system is the octantal component due to the finite spacing s, which has the maximum value of $23.5\ (s/\lambda)^2$ degrees for a wave of wavelength λ arriving horizontally.

The discussion in the following sections will be confined to the causes and elimination of those errors due to inequalities in the aerials and associated circuits and to unbalance in the aerial transformer or goniometer.

3.1. *The balance of the H-Adcock aerial pair*

Fig. 7 (a) represents an aerial pair of an elevated H-Adcock system. APB and CQD are the two limbs of the system and EM is the vertical conducting column. The upper and lower halves of the dipoles have capacitances C_2 and C_3 respectively to earth, and C_4 is the capacitance between the halves. C_1 and C'_1 are the capacitances of the limbs to the top of the column M and are composed of the capacitance of the aerials and feeders to M and of the lead-in wires and transformer primary (or goniometer field-coil) L_1 to the screening box which houses them and is electrically connected to M. On account of the composite nature of the capacitances C_1 and C'_1, they are not assumed necessarily to be equal, whereas accurate geometrical symmetry of aerials will ensure equality of corresponding capacitances C_2, C_3 and C_4 for the two sides. C_d is the differential capacitor used for balancing the capacitances of the limbs to M.

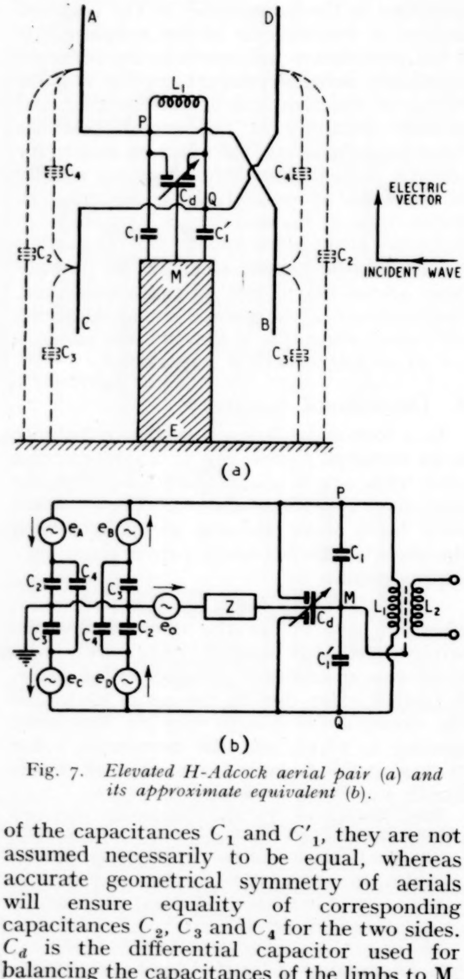

Fig. 7. *Elevated H-Adcock aerial pair (a) and its approximate equivalent (b).*

The equivalent circuit is shown in Fig. 7 (b) in which e_A, e_B, e_C and e_D are the aerial e.m.fs. induced by the incident wave while e_0 and Z are the equivalent induced e.m.f. and the impedance of the vertical column.

Theoretically, a vertical conductor at the centre of an H-type aerial system can give rise to no errors providing mechanical and electrical symmetry is preserved. Serious trouble can arise, however, when these requirements are not met, particularly near resonant frequencies of the vertical conductor.

The effects have been noted with H-type aerial systems having a long central column, but in the case of aerial circuits not involving tuning of the primary of the aerial transformer, it has been possible to obtain good aerial balance right through the resonance of the column by very careful adjustment of the aerial balancing capacitors and also by ensuring that the aerial transformer is constructed as symmetrically as possible.

The object of balancing the aerial pair is to arrange that when the plane of the pair is normal to the plane of propagation (and thus $e_A = e_D$ and $e_B = e_C$) there shall be no resultant e.m.f. in the secondary (or search coil) L_2. The problem can be resolved into two parts :

(i) That the goniometer or transformer L_1, L_2 shall be free from unbalance in order that the secondary e.m.f. shall be zero when the p.d. across the primary is zero, whatever the mean potential of the primary with respect to the screening (i.e., to M.).

(ii) That the aerial and column e.m.fs. shall be balanced to give zero primary p.d.

The first aspect of the problem has been discussed in another paper[3] with particular reference to the causes and measurement of unbalance in goniometers. It is possible to construct screened transformers for use in the line-coupled elevated H-system which have close coupling and yet possess a sufficiently high degree of balance. The design of tightly-coupled transformers to satisfy this requirement usually requires some form of concentric windings with provision for axial adjustment of the relative positions of the windings. With aerial circuits employing a tuned primary arrangement, however, it has proved quite impossible to arrange that the capacitances of the two sides of the tuning capacitor to earth remain balanced accurately enough to ensure that errors in the neighbourhood of the column resonance shall be small.

The second part of the problem will now be discussed by reference to Fig. 8. In Fig. 8(a) the aerial e.m.fs. and capacitances have been transformed and the condition for balance is seen to be such that the potentials of P and Q are equalized by adjustment of C_d ; i.e., that C_1 and C'_1 are equalized. The condition for balancing the column e.m.f. is seen from Fig. 8(b) to correspond to the balancing of an equal ratio capacitance bridge excited by e_0, and again adjustment of C_d to give equality of C_1 and C'_1 is required.

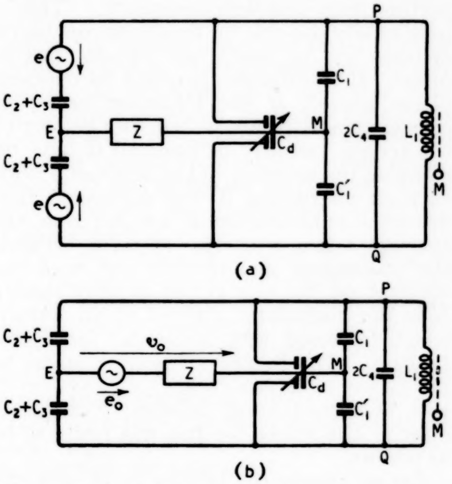

Fig. 8. *Simplified equivalent circuits showing the method of balancing : (a) represents the equivalent circuit for the balancing of the aerial e.m.fs. (plane of aerial system normal to plane of propagation* $\therefore e_A = e_D$; $e_B = e_C$) *and (b) shows equivalent circuit for the balancing of the column e.m.f.*

Since, therefore, the balancing adjustment of C_d is the same for both types of e.m.f. when the dipole capacitances are equal, it can readily be seen that this adjustment should hold over a band of frequencies and for any ratio of column and aerial e.m.fs. This is found to be the case in practice and the balancing process need only be carried out at the frequency where it is most critical, namely at the column resonance frequency or at the lowest frequency in the band.

It will be appreciated that all the problems of balance or symmetry become increased at the quarter-wave column resonance, at which the column e.m.f. is the predominant source of trouble, for it will be very much greater than the aerial e.m.f. For example, if the resistance or conductance losses in the capacitance arms of the bridge in Fig. 8(b) are unequal they will produce an unbalance whose effect is in quadrature with that of capacitance unbalance and cannot therefore be taken up by readjustment of C_d. If the causes of the losses cannot be removed, they may be at least partially remedied by connecting suitable low-wattage composition resistors between either P or Q and M. Similarly at the column resonance, screening of the box at the top of the column housing the aerial circuits becomes particularly important, and it is also necessary to locate the tapping points P and Q accurately at the electrical centre of the feeders.

Examination of Fig. 8(b) will show that the balance of C_d to make the potentials of P and Q equal does not also reduce their mean potential to that of M ; i.e. of the screening.

Theory and experiment show that in typical cases the mean potential may be several hundred times the maximum directional p.d. across PQ. For instance, it has been found that for one particular tuned-aerial system, with a long down lead, a misadjustment of the aerial balancing capacitor which, at a frequency of 12 Mc/s, produced a reciprocal error of only 3 degrees and a slight flattening of bearings, caused a reciprocal error of over 30 degrees at the column resonance frequency, 15 Mc/s. It will be realized how difficult is the problem of designing a tuning capacitor maintaining the symmetry of its balance to earth with sufficient accuracy.

The difficulty of balancing always increases as the ratio (s/λ) of spacing to wavelength decreases and in practice this sets a lower limit to this ratio in the design of compact systems. Experience shows that (s/λ) should not be less than 1/50 and may need to be larger if the column resonance frequency occurs near the lower end of the working frequency band.

Difficulties due to column resonances are found, in practice, to be confined to frequencies near the first one or two resonant modes of the column. For this reason the effects are of greatest importance in the high-frequency band, practical considerations making it improbable that they will occur as either very low or very high frequencies.

Since the column e.m.f. e_0 is the important one in balancing the aerial system, excitation of the column by a local oscillator provides a useful method of carrying out the adjustment when the system is erected on a bad site in which local reradiation is pronounced. A suitable method of excitation is by means of a screened toroid which is arranged to embrace the base of the column ; a current in the toroid will excite a substantially lumped e.m.f. in the column. The unwanted electric field at the Adcock aerials due to the toroid will be sufficiently small to be negligible.

3.2. *Errors due to inequality between the aerial pairs in a four-aerial system*

If the circuits between the aerial pairs and the field-coils of the goniometer are not identical, a quadrantal error will be produced for inequality of amplitude and a quadrantal flattening of bearing for inequality of phase.

In the directly-coupled system the only source of inequality is in the reactance of the aerial and field-coil circuits. A 3.5 % inequality of reactance will give a 1-degree quadrantal error ; but since the system is used in the region of primary resonance the tolerances in the aerial and feeder capacitance or field-coil inductance will be more stringent.

In practice it is necessary to equalize the capacitance and inductance to an accuracy of about 1 % if the quadrantal error is to be negligible at the edge of the band near the primary resonance.

In the line-coupled system the problem is more complicated, for apart from inequalities in the transmission lines, any inequality of the impedances presented by the secondaries of the aerial transformers will be transformed to an extent which depends markedly on frequency. The quadrantal error in this system is greatest at the frequencies where the transmission lines, together with their terminations, resonate, and in practice one or more of the resonances will occur in the operative band. It is found in practice that these errors cannot be eliminated by striving for equality in the corresponding components of the two channels, for a very high degree of equality is required, and even if it could be attained it is doubtful whether the secular stability would be adequate.

The most satisfactory remedy is to connect across each transmission line at its base where it is connected to the goniometer field-coil, a resistor of suitable value between about Z_0 and $3Z_0$. The resistor introduces heavy damping at those frequencies where there would otherwise be a series resonance at the lower end of the line and the quadrantal error at these frequencies due to inequalities is thereby considerably reduced. The sensitivity is also reduced at these frequencies, but since in the absence of damping the sensitivity would tend to be enhanced, the result is usually a more uniform overall performance.

3.3. *Effect of proximity of observer in a rotating-aerial system*

With rotating-aerial systems, the presence of the operator usually constitutes a departure from complete symmetry. This is particularly the case for small compact aerial systems and in nearly all portable instruments. If bearings are sharp and the operator places himself at the minimum he is, of course, symmetrically placed with respect to the aerials, and may be assumed to have no effect on the accuracy of the bearings. With such an instrument it was discovered that when the minimum signal was not zero, the bearings taken with the operator between the transmitter and receiver and with the operator on the side of the receiver remote from the transmitter, differed by an amount which increased with increasing flatness of bearing. (The flatness of bearing might be due to any of several causes : e.g., site error, lack of correct aerial balance, polarization error.) The reason for this was found to be as follows.

Let us assume the bearing to be perfectly sharp so that an exact zero of signal exists, and that the operator places himself symmetrically with respect to the aerials. If, while remaining stationary himself, he swings the aerials out of the minimum, he will observe a signal which increases as the aerials are rotated out of the position of the minimum. Part of this signal is the normal signal due to the phase difference introduced between the e.m.fs. in the two aerials, but part is due to the fact that the aerial balance has been disturbed through one aerial now being nearer the observer than the other. This second part of the signal is, to a first approximation, in phase quadrature with the other, and its phase will be reversed if the operator moves round through 180 degrees since it is now the other aerial which is nearer him. The sign of this part of the signal also depends on whether the aerials are rotated clockwise or anti-clockwise from the minimum.

Now suppose there is no point of zero signal, because for some reason or other there exists a component of pick-up in phase quadrature with the wanted pick-up, and suppose the aerials to be placed in the position of the minimum by some means with the operator removed. Now replace the operator at the minimum and let him remain stationary. As he rotates the aerials, first clockwise and then anti-clockwise, he will produce an unbalance current in the aerials due to his presence (in phase quadrature with the pick-up due to phase difference between the e.m.fs. in the two aerials as described above) which will aid or oppose, according to the sense of rotation of the aerials, the phase quadrature signal existing at the minimum. He will, therefore, actually observe the minimum to be displaced from the position in which he found the aerials and an error in bearing will result whose sign will depend on whether he has placed himself at the minimum on the side of the receiver nearer the transmitter or at the opposite minimum.

The effect could be avoided by arranging that the operator remained rigidly attached to the aerial system in a position symmetrical with respect to the aerials and rotated with it when observing a bearing, or by placing on the opposite side of the aerial system to the operator a " dummy " operator designed to have the same influence on the aerial impedances when the aerials move up to it as does the operator.

The greater the aerial spacing and the shorter the wavelength the less is the effect, but as an example of its seriousness it may be mentioned that for an aerial system with a spacing of 1.2 metres and at a wavelength of 50 metres (6 Mc/s), for bearings which were flat to the extent of requiring a swing of ±5 degrees in order to determine the minimum, a bearing error of 3 degrees or 4 degrees was observed. This is serious when one considers that bearings taken on ionospheric signals are usually rather flat, swings exceeding ±5 degrees being by no means rare.

These effects show that the hitherto accepted principle that no errors due to the observer will result if he places himself at what he observes to be the minimum, is not sound and may lead to substantial bearing errors.

4. Polarization Error

Since the Adcock systems here under discussion have vertical aerials, polarization error arises from undesired reception of the wave component having its electric vector horizontal.

A fundamental cause of polarization error in an otherwise ideal H-system is proximity to the ground, which introduces an asymmetry as was first demonstrated by Barfield[1]. The horizontally polarized field acting at the feeders (whether enclosed by a metallic " screening " tube or not) will produce a resultant output due to the inequality of the capacitances of the upper and lower portions of the aerials. Furthermore, the mere existence of the ground as a reflecting surface below the aerials, gives rise to polarization error in any H-type aerial system, however nearly symmetrical and carefully balanced. The mechanism of the effect is as follows. The horizontal electric field acting on the horizontal feeder wires or on any screen surrounding them sets up longitudinal currents in these horizontal conductors. In an electrically-balanced aerial system these currents should not give rise to any resultant circuital current through the receiving impedance, but the horizontal conductors constitute a horizontal doublet, which has an image in the ground, with the result that the secondary field, although arising from a horizontal doublet, has vertical components at the aerials. Thus an e.m.f. is induced in the vertical aerials which originally arose from the horizontal electric force. The effect is not to be confused with that mentioned by Barfield[1] due to the upper and lower dipoles having different impedances to ground, but represents a fundamental limitation to the perfection of the H-type of Adcock direction finder.

The magnitude of the error should, according to simple theory, vary approximately as the cube of the ratio of the aerial spacing to height above the ground. Calculations made for a system described elsewhere suggest that this effect could only account for a fraction of the polarization error actually observed, and tests to discover how the error varied with height of the aerial system tended to show that here also, the effect under discussion could not be the main cause of the error. It is of importance, however, to note that the presence of the earth below an H-type Adcock direction finder prevents it from having zero polarization error, however nearly symmetrical and balanced the aerial system.

4.1. *Effect of a metal mass near the lower halves of the dipoles*

The effects hitherto described represent inherent sources of polarization error in an otherwise ideal H-system due to its proximity to the ground. Experiment has, however, shown that in practical systems where there is a receiver or screening box between or near the lower halves of the dipoles the presence of the mass of metal can give rise to polarization error for these reasons :

(i) the presence of the metal causes the capacitance of the upper and lower halves of the aerials to be unequal ;

(ii) horizontal currents are induced in the mass of metal by the horizontal electric field, causing the mass to behave like a short horizontal doublet, which, because of its position relative to the aerials, reradiates into these aerials energy originally coming from the horizontal electric field ;

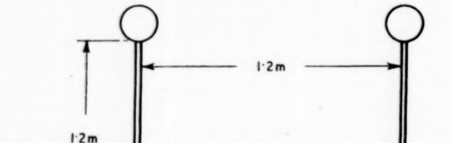

Fig. 9. *Typical H-Adcock portable direction finder (to scale).*

(iii) circulating currents induced in the metal mass by the horizontal electric field reradiate into the aerials.

A simple cure for (i) above would be to place a similar mass of metal between the upper halves of the aerials but this would not reduce effects (ii) or (iii). The two masses being at different heights would be in different horizontal electric fields and the reradiation from them into the aerials would not cancel.

That the presence of the receiver was responsible for a substantial portion of the error in a typical system (Fig. 9) was proved by lowering the receiver to the ground, extending the central column so as to leave the aerials at the same height above ground. A reduction of 30% to 50% of the error was the result. Further, when a bulkier receiver case was used the polarization error was found to be very much greater than in the original instrument. This effect, due to the receiver, was enhanced because of the very small physical aerial spacing used. With greater spacing, the effect of the receiver would be less, but sufficient effect was found to show that the practice of placing the receiver close to the lower halves of the aerials is not a good one.

4.2. *Outstanding problems*

As a result of an examination of the performance of several experimental H-Adcock direction finders it appears that all the sources of polarization error have not yet been discerned. The causes enumerated in this section are certainly responsible for a significant part of the polarization error in certain direction finders but they have been found to be inadequate to account for the whole of the errors. Further work is necessary particularly on the manner in which the presence of the ground destroys the inherent accuracy of the ideal H-aerial system before the high performance of which it should be capable can be attained.

5. Conclusions

This paper sets out certain features which are of importance in the design of H-type Adcock direction finders for the high-frequency band (3–30 Mc/s approximately). Certain of these are relevant at other frequencies as well. The conclusions arrived at may be summarised as follows :—

(*a*) *Sensitivity*

In the frequency band referred to, the dimensions of most practical H-type systems are small in comparison with the wavelength of operation. In these circumstances maximum sensitivity is obtained by :—

(i) Maintaining as high a ratio as possible of aerial capacitance to total aerial-circuit parallel capacitance ;

(ii) Maintaining a high ratio of effective to physical length of aerials, by end-loading of the aerials where practicable ;

(iii) Arranging for the primary inductance to resonate with the total aerial-circuit capacitance at a frequency just above the highest frequency in any particular band (in practice, at about 1.25 times the highest frequency) ;

(iv) Tuning the search coil of the goniometer or secondary of aerial transformer whenever possible and avoiding further intermediate links ;

(v) Arranging that the total working frequency band is covered in a series of sub-ranges each of the order of 1.5 to 1 in frequency by suitable switching or otherwise interchanging aerial transformers or goniometers.

(*b*) *Directional accuracy*

(i) The presence of a vertical column at the centre of an H-type aerial system comprising the down leads and any metallic supporting structure or mast enhances the difficulties of aerial balancing in the neighbourhood of frequencies for which the column is resonant. In particular, the aerials and associated coupling circuits require to be very accurately balanced at the lowest natural frequency of the column occurring in the working band of the direction finder.

(ii) In rotating systems in which the aerials come into proximity with the observer (e.g., portable H-type direction finders) errors can be caused by the presence of the observer even when he places himself at what appears to him to be the minimum. These errors only occur if the minimum is flat (due to site error, or some other such cause) and are of importance when the ratio of aerial-spacing to wavelength is small. In certain circumstances it may be necessary to compensate for the presence of the observer by placing an equivalent dummy on the opposite side of the direction finder.

(*c*) *Polarization errors*

(i) There is no reason to doubt that, in free space, an idealized H-type aerial, consisting of two equal dipoles connected to a very small receiver at the centre, would be free from polarization error. In practice, the presence of the earth, and in some circumstances the presence of the receiver, introduce polarization errors.

(ii) One essential source of polarization error due to the proximity of the ground is of interest because it represents a fundamental limitation of the H-system. It is due to the image in the ground of the horizontal conductor formed by the feeder lines (and their screening tube, if any). This horizontal conductor, excited by any horizontal electric field in the wave produces, along with its image in the ground, secondary vertical electric fields at the aerials. The resulting polarization errors are however only important when the height of the centre of the aerial system above ground is small compared with the aerial spacing.

(iii) The presence of the mass of metal constituted by the receiver and its housing has been shown to be a cause of polarization errors. In practice the receiver should be placed as far below the aerials as possible, preferably on the ground.

(iv) The disturbing influence of the ground on the symmetry of the H-aerial system is not yet fully understood. It is not yet possible to account for the whole of the polarization error in existing systems.

6. Acknowledgments

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COMMON-CATHODE AMPLIFIERS

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SUMMARY. The results of a previous paper are applied to the discussion of the optimum adjustment and performance of a common-cathode amplifier (triode or pentode) at about 50 Mc/s.‡

The most important characteristics are the noise factor N, the input conductance G_i (which should both be small), and the effective bandwidth, which should be large and is taken to be expressed by the ratio ("A-ratio") of the minimum to maximum gain within a band defined by an ideal filter. These three conditions are incompatible; it is assumed that optimum adjustment is that which gives minimum N for a prescribed A-ratio (which is taken to be $\frac{1}{2}$), regardless of G_i and the absolute gain; but some remarks are offered on the modifications necessary if these last two characteristics are important.

In valves designed for operation near 50 Mc/s transit-time effects (in the usual sense) are inappreciable at that frequency; but the effects of the inevitable inductance in the electrode leads are appreciable. A complete discussion of these effects would be almost intolerably laborious, though simple in principle. Accordingly it is assumed that the effects are small; that the amplifier is adjusted to optimum performance in the absence of the inductances; and that inductances having one (pentode) or two (triode) prescribed sets of values are then introduced. The difference produced by their presence can then be ascertained with considerable, but not prohibitive labour. It is discussed how far conclusions indicated by this method have practical significance.

The main conclusions concerning the triode are given in Sections 5 and 6 with Tables I and II; those concerning the pentode in Sections 8 and 9 with Tables III and IV. The triode and pentode are compared in Section 10. The appendix contains formulae relating to the effect of the inductances that would complicate the text unnecessarily.

1. Introduction

In this paper we apply the results of a previous paper,[1] here denoted by C, to problems arising in common-cathode negative-grid amplifiers operating at about 50 Mc/s. The paper denoted in C by B will again be so denoted. Complete knowledge of these papers is assumed in what follows.

At 50 Mc/s the inevitable inductance in the electrode leads has an appreciable effect. A completely general discussion of the problem therefore involves algebraical formulae which, though straightforward, are intolerably complex. Accordingly in much of the general discussion we shall assume that these inductances are all zero. Having arrived at practical conclusions valid on that assumption, we shall examine the effect of introducing inductances to which particular numerical values are assigned. Much of the algebra necessary in this second stage will be relegated to an appendix or omitted altogether. The practical significance of the results attained by this procedure will be discussed later.

The question arises whether, if the lead inductances have to be taken into account, the transit time of the electrons can be ignored as it was in C. Transit-time effects are dealt with in another paper.[2] Our conclusion is that, at 50 Mc/s in valves designed for that frequency, the effects usually associated with transit time, which involve the square and higher powers of the transit angles, are inappreciable. On the other hand, the first order effect, proportional to the transit angle, appears as the well-known change in the inter-electrode capacitances with the electron current. If the values assigned to these capacitances are those measured with the actual electron current flowing, then no further account need be taken of transit time at this frequency. Accordingly we shall continue to ignore transit time, but in all numerical work we shall use capacitances so measured.

2. Common-cathode Triode—General Formulae

We start with a common-cathode amplifier using a triode as shown in Fig. 1. L_1, L_2, L_3 are the inductances in the cathode, grid, and anode leads respectively. Y_u is the admittance looking into the output of a

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MS accepted by the Editor, January 1947.

‡ The use of the word "common" in this connection denotes that electrode which is common to input and output circuits.

potential transformer by which the input circuit is fed, E_u the transformed signal voltage effectively in series with Y_u and γ_u the conductance in parallel with Y_u that represents the transformer loss. (See C, Section 7). C_u is the capacitance to earth of the input pin of the valveholder. Y_v is the load admittance across which the output voltage is taken, and C_v is the capacitance of the valveholder at the output. Y_{45} is the admittance of a mainly inductive impedor introduced in order to tune out the grid-anode capacitance C_{23} and thus reduce the risk of oscillation ; it includes a blocking capacitor of negligible reactance and any earth capacitance associated with it is included in C_u or C_v. G_{12} represents a conductive leak between cathode and grid, such as is often found in r.f. triodes. When the inductances are zero, it is directly in parallel with γ_u ; in this case we shall write $\gamma_u + G_{12} = \gamma'_u$.

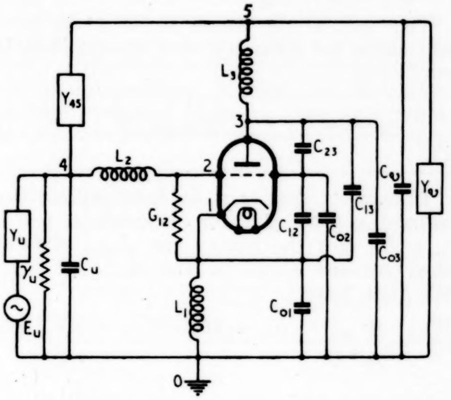

Fig. 1. *Circuit for a triode including lead inductances.*

Y_{45} must be connected across the outer ends of L_2, L_3, because the inner ends are inaccessible. It follows that L_2 is common to the external circuits (0,2), (2,3), and L_3 to the circuits (2,3), (0,3). The conditions in which the formulæ of B apply are therefore violated, if we recognise only terminals 0,1,2,3. But if we recognise terminals also at 4,5, which, like the earth terminal, are not electrodes, these conditions are fulfilled ; the formulæ of B can still be used, if it is remembered that there are now 6 terminals and therefore a determinant of 25 terms. Since the terminals 4,5 receive no electron current, and their potentials do not directly affect that current, α_4, α_5, g_4, g_5 are all zero. Further there is no external admittance between 4 and 1 or 3, or between 5 and 1 or 2. Hence the elements (41), (51), (52), (43), (14), (34), (15), (25) are all zero and

$$\Delta = \begin{vmatrix} (11) & (21) & (31) & 0 & 0 \\ (12) & (22) & (32) & (42) & 0 \\ (13) & (23) & (33) & 0 & (53) \\ 0 & (24) & 0 & (44) & (54) \\ 0 & 0 & (35) & (45) & (55) \end{vmatrix} \qquad (2.1)$$

where, writing C_n for the sum of the capacitances one of whose suffixes is n,

$$(11) = 1/j\omega L_1 + (11)', \text{ where } (11)' = g_2 + g_3 + G_{12} + j\omega C_1$$
$$(21) = -g_2 - G_{12} - j\omega C_{12}$$
$$(31) = -g_3 - j\omega C_{13}$$
$$(12) = -G_{12} - j\omega C_{12}$$
$$(22) = 1/j\omega L_2 + (22)', \text{ where } (22)' = G_{12} + j\omega C_2$$
$$(32) = -j\omega C_{23}$$
$$(42) = -1/j\omega L_2$$
$$(13) = -g_2 - g_3 - j\omega C_{13}$$
$$(23) = g_2 - j\omega C_{23}$$
$$(33) = 1/j\omega L_3 + (33)', \text{ where } (33)' = g_3 + j\omega C_3$$
$$(53) = -1/j\omega L_3$$
$$(24) = -1/j\omega L_2$$
$$(44) = 1/j\omega L_2 + (44)', \text{ where } (44)' = G_u + G_{45} + \gamma_u + j(B_u + B_{45} + \omega C_u + \omega C_{45})$$
$$(54) = -G_{45} - jB_{45} - j\omega C_{45}$$
$$(35) = -1/j\omega L_3$$
$$(45) = (54)$$
$$(55) = 1/j\omega L_3 + (55)', \text{ where } (55)' = G_v + G_{45} + j(B_v + B_{45} + \omega C_v + \omega C_{45}) \quad .. \quad (2.2)$$

The quantities for which formulae are given in C, and on which we shall base our discussion, are the noise factor N, the signal gain A and the input conductance G_{04i}. The sources of noise are the conductors G_u, γ_u, G_{12}, G_v, G_{45}, and the electron current represented by $G_{13\sigma}$. Hence, starting from C(10.6), substituting for $R_s|A|^2$ from C(11.2), and replacing symbols ρ by δ—for this is merely ignoring the common factor $1/\Delta$—we have

$$N = 1 + \frac{\int\{\gamma_u|\delta_{05,04}|^2 + G_{12}|\delta_{05,12}|^2 + G_{45}|\delta_{05,45}|^2 + G_v|\delta_{05,05}|^2 + aI_0\Gamma^2|\delta_{05,13}|^2\}df}{\int G_u|\delta_{05,04}|^2 df} \quad .. \quad (2.3)$$

where $\delta_{05,04} = \Delta_{54}$; $\delta_{05,12} = \Delta_{52} - \Delta_{51}$;

$$\delta_{05,45} = \Delta_{55} - \Delta_{54} \text{ ; } \delta_{05,05} = \Delta_{55} \text{ ;}$$
$$\delta_{05,13} = \Delta_{53} - \Delta_{51} \quad .. \quad .. \quad (2.4)$$

$$|A|^2 = G_u G_s \left|\frac{\delta_{05,04}}{\Delta}\right|^2 \quad .. \quad (2.5)$$

where G_s is the internal conductance of the signal generator, and

$$\Delta = (53)\Delta_{53} + (54)\Delta_{54} + (55)\Delta_{55} \quad (2.6)$$

(the terms in $1/j\omega L_3$ from $(53)\Delta_{53}$ and $(55)\Delta_{55}$ cancel each other).

$$G_{04i} = \text{real part of } \Delta/\Delta_{44} \quad .. \quad .. \quad (2.7)$$

3. Triode without Lead Inductances

We now assume that $L_1 = L_2 = L_3 = 0$. In order to obtain the formulae for this case, we add the fourth row of Δ in (2.1) to the second, and the fifth to the third ; multiply the resulting first, fourth, and fifth rows by $j\omega L_1$, $j\omega L_2$, $j\omega L_3$ respectively ; and equate to zero all terms with an L as a factor. If x^3 means $j\omega L_1 \,.\, j\omega L_2 \,.\, j\omega L_3$, we find

$$x^3\Delta = \begin{vmatrix} (22)_0 & (32)_0 \\ (23)_0 & (33)_0 \end{vmatrix} \quad .. \quad .. \quad (3.1)$$

where

$$(22)_0 = (22)' + (44)' = G'_u + jB'_u$$
$$(32)_0 = (32) + (54) = -G_{45} - jB'_{45}$$
$$(23)_0 = (23) + (45) = g'_2 - jB'_{45}$$
$$(33)_0 = (33)' + (55)' = G'_v + jB'_v$$
$$.. \quad .. \quad .. \quad (3.2)$$

and $G'_u = G_u + \gamma_u + G_{12} + G_{45}$;

$$B'_u = B_u + B_{45} + \omega(C_u + C_2 + C_{45})$$
$$G'_v = G_v + G_{45} + g_3 \text{ ;}$$
$$B'_v = B_v + B_{45} + \omega(C_v + C_3 + C_{45})$$
$$B'_{45} = B_{45} + \omega(C_{23} + C_{45})$$
$$g'_2 = g_2 - G_{45} \quad .. \quad .. \quad .. \quad (3.3)$$

The vanishingly small factor x^3 in (3.1) will cause no difficulty as it always cancels out.

In using (2.3) to (2.7), we must replace the suffixes 4, 5 by 2, 3, for terminals 2, 4 and 3, 5 are now the same ; the $\Delta_{(mn)}$ of (2.1) then become equal to the $\Delta_{(mn)_0}$ of (3.1). We thus have

$$N = 1 + \frac{\int\{\gamma'_u|g'_2 - jB'_{45}|^2 + G_{45}|G'_u + g'_2 + jB'_u - jB'_{45}|^2 + (G_v + aI_0\Gamma^2)|G'_u + jB'_u|^2\}\,df}{\int G_u|g_2' - jB'_{45}|^2\,df}$$
$$.. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.4)$$

$$|A|^2 = G_u G_s \frac{g'^2_2 + B'^2_{45}}{\{G'_u G'_v + G_{45}g'_2 - B'_u B'_v + B'^2_{45}\}^2 + \{G'_u B'_v + G'_v B'_u + (g'_2 - G_{45})B'_{45}\}^2}$$
$$.. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.5)$$

$$G_{04i} = G_{02i} = G'_u + \text{real part of } \frac{(g'_2 - jB'_{45})(G_{45} + jB'_{45})}{G_v' + jB'_v} \quad .. \quad .. \quad .. \quad .. \quad (3.6)$$

4. The Problem of Adjustment

Of the quantities involved in (3.4) — (3.6), g_2, g_3, I_0, Γ^2 and the minimum values of G_{12} and of all the C terms are determined by the valve used ; they and G_s may be taken as given. On the other hand Y_u, Y_v, Y_{45} are in principle at our disposal, except for some limitation on the approach of G_u, G_v, G_{45} to zero. γ_u is determined by the components of Y_u. The problem to be discussed is how the disposable values should be adjusted so as to give optimum performance. In discussing it we shall assume that the amplifier includes an ideal filter on the output side, having a cut-off at $f = f_0 \pm \delta f$, but constant transmission between the cut-off frequencies ; the integrals have then to be taken only between the cut-off frequencies.

One feature of optimum performance is a small N. N cannot be reduced to unity, unless G_{45} and γ'_u are both zero ; but by making each B' zero at f_0, N can be reduced to a minimum at that frequency by a suitable choice of G_u, which is not zero. (See C, Section 12). But other features are a flat response curve and a low input conductance ; the G_u that gives minimum N may give too great a variation of $|A|^2$ within the band and too great a G_{04i}. The adjustment has therefore to be a compromise determined by the exact conditions in which the amplifier is to be used. In order to have a definite problem, we shall suppose that the bandwidth is fixed by prescribing a lower limit to the ratio of the minimum to the maximum value of $|A|^2$ within the band, and that the input conductance is relatively unimportant. The main problem is then to find the minimum value of N that is consistent with the prescribed value of the said ratio, which will be termed the A-ratio; the prescribed value will be taken to be ½.

Other simplifying assumptions will be made. First, that each B is the susceptance of a tuning inductor L in parallel with a capacitance C, which is the self-capacitance of the tuning inductor. In the case of B_u and B_v, this capacitance can be merged in C_u or C_v, and thus requires no separate recognition. In the case of B_{45}, it can be merged in C_{23} if, but only if, $L_2 = L_3 = 0$; accordingly it will be denoted separately as C_{45}. The losses in the tuning coils are part of G_u, G_v, G_{45}.

Secondly, it is assumed that the inductors are chosen so that each B' is zero at f_0. $|A|^2$ will then have its maximum near, but not necessarily exactly at, f_0. This does not necessarily give precisely the optimum adjustment; but, in at least one commonly used empirical method of adjusting to overall optimum conditions, departures from the assumed values will be small. If

$$y = f/f_0 - f_0/f \quad \ldots \quad \ldots \quad (4.1),$$

we then have at any frequency

$$\begin{aligned} B'_u &= y\omega_0(C_u + C_2 + C_{45})\,; \\ B'_v &= y\omega_0(C_v + C_3 + C_{45})\,; \\ B'_{45} &= y\omega_0(C_{23} + C_{45}) \quad \ldots \quad (4.2) \end{aligned}$$

Thirdly, it is assumed that each of G_u, γ_u, G_{12}, G_v, G_{45} may be taken as independent of frequency within the band. This cannot be really true if any considerable part of these conductances represents the losses in a tuning coil of approximately constant Q; it might introduce appreciable error into the results for a really wide-band amplifier; but in view of other uncertainties, the assumption is likely to be sufficient if $\delta f/f_0$ does not exceed 0.1. It is also assumed that γ_u is independent of G_u; this will be very nearly true if the input "transformer" is an inductor across a small portion of which the signal generator is tapped.

With these assumptions (3.4), (3.5) become

$$N = 1 + \frac{\gamma'_u}{G_u} + \frac{G_{45}\{(G'_u + g'_2)^2 + \overline{y^2}\omega_0^2(C_u + C_2 - C_{23})^2\} + (G_v + aI_0\Gamma^2)\{G'^2_u + \overline{y^2}\omega_0^2(C_u + C_2 + C_{45})^2\}}{G_u\{g'^2_2 + \overline{y^2}\omega_0^2(C_{23} + C_{45})^2\}} \quad \ldots \quad (4.3)$$

$$|A|^2 = G_uG_s \frac{g'^2_2 + y^2\omega_0^2(C_{23} + C_{45})^2}{\{G'_uG'_v + G_{45}g'_2 - y^2\omega_0^2[(C_u + C_2 + C_{45})(C_v + C_3 + C_{45}) - (C_{23} + C_{45})^2]\}^2 + y^2\omega_0^2\{G'_u(C_v + C_3 + C_{45}) + G'_v(C_u + C_2 + C_{45}) + (g'_2 - G_{45})(C_{23} + C_{45})\}^2} \quad \ldots \quad (4.4)$$

In (4.3) $\overline{y^2}$ is the mean of y^2 throughout the band; it is easily found to be given by

$$\overline{y^2} = \frac{4}{3}\left(\frac{\delta f}{f_0}\right)^2 + \sum_{n=2}^{\infty}\left(\frac{\delta f}{f_0}\right)^{2n} \quad \ldots \quad (4.5)$$

It may be remarked incidentally that, since the width of the band enters only through $\overline{y^2}$, which is so easily calculated, the usual practice of identifying N with its value at the centre of the band is unnecessary.

5. Adjustment of Triode without Lead Inductances

It may be seen that the optimum values of the capacitances are their minimum values, and that this is especially true of C_2, the total grid capacitance. For a large A-ratio requires that $G_u'G'_v$, the term in the denominator of (4.4) independent of frequency, should be large compared with any frequency-dependent term, of which the largest turns out in practice to be $y\omega_0G'_vC_2$. Since we shall find it necessary to choose G'_u larger than that which gives minimum N, it is desirable to decrease G'_u, and therefore to decrease C_2. Indeed it may be said here that a general result of our inquiry into the triode and pentode is to show that the first requisite for a good performance in the optimum adjustment is a small grid capacitance.

For the rest, sufficient increase of any of the conductances will make the A-ratio exceed ½ (with some reservations noted presently). The question is which sufficient increase produces the smallest N. G_{45} can be excluded on practical grounds; actually it can be excluded on all other grounds and it should be the minimum set by the Q of the tuning coil L_{45}. In order to examine the others, it is most convenient to assume numerical values for all the fixed constants; for they are not likely to vary greatly in practice. The values in Table I are those measured on a plane-parallel triode, developed for use at a mean frequency of 45 Mc/s with a bandwidth of 4 Mc/s. Values are also given for G_s, G_{45}, γ'_u. G_s affects only the absolute gain, but not N or the A-ratio; it is therefore unimportant. G_{45} corresponds to a Q of about 50 in the tuning coil L_{45}. The smaller γ'_u assumes $G_{12} = 0$, so that $\gamma'_u = \gamma_u$, and that the signal generator is connected to a tapping on an input inductor whose Q is about 50; the larger assumes that G_{12} is equal to the γ_u represented by the smaller.

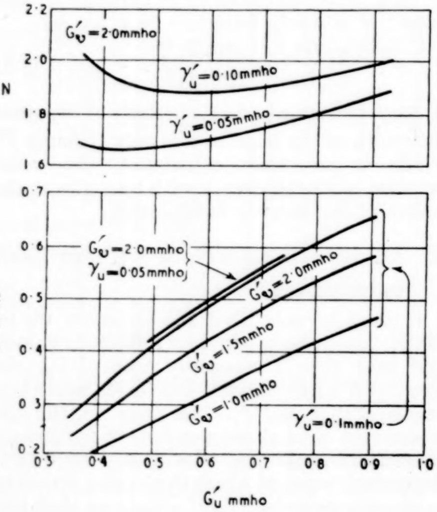

Fig. 2. *Variation of N and A-ratio with generator coupling for the triode.*

The only variables are now G_u and G_v. We assume a series of values for the latter, and calculate how N and the A-ratio vary with the former. (It is more convenient to work with G'_u, G'_v rather than G_u, G_v). The results are shown in Fig. 2, where the abscissa is G'_u, the ordinate N in the upper part of the figure, the A-ratio in the lower part. It should be noted that both N and $|A|^2$ are always given as ratios, not in decibels. Since G_v is always much less than $G_{13\sigma}$, N varies little with G'_v; accordingly N is plotted only for $G'_v = 2$ mmho. Similarly, since the A-ratio, plotted against G'_u, varies little with γ'_u, only part of one of the A-ratio curves is plotted with the smaller (and less probable) value of γ'_u.

Since, for a given G'_u, N increases with γ'_u while the A-ratio is nearly independent of it, γ'_u should be as small as possible; it is highly desirable to reduce the leak within the valve. On the other hand, though N is almost independent of G'_v, the A-ratio increases rapidly with G'_v when G'_v is small, but much less rapidly when it is large. Since a large G'_v means a small absolute gain and some increase in N, it is undesirable to increase it beyond the point where increase produces a marked beneficial effect. Accordingly G'_v should be about 2 mmho. At this value it is impossible to choose G'_u so that N is a minimum and the A-ratio is $\frac{1}{2}$ simultaneously. But, when $\gamma'_u = 0.1$ mmho, the difference between N when the A-ratio is $\frac{1}{2}$ and minimum N is much less than 0.1, and is hardly important. When $\gamma'_u = 0.05$ mmho, the difference is rather greater; but then it is impossible to attain minimum N however large G'_v is. So, if N and the A-ratio alone matter, there is no reason to depart from $G'_v = 2$ mmho. Then

$$G'_u = 0.62 \text{ mmho};$$

$$G_u = 0.50 \text{ mmho, if } \gamma'_u = 0.1 \text{ mmho};$$
$$G_u = 0.55 \text{ mmho, if } \gamma'_u = 0.05 \text{ mmho}.$$

If $\gamma'_u = 0.1$ mmho, then at f_0, $N = 1.83$; $|A|^2 = 170.7$; A-ratio $= 0.50$; $G_{04i} = 0.69$ mmho. It is to be observed that, in this optimum adjustment, G_u is *not* $\frac{1}{2}G_{04i}$, as it would be if G_u were "matched" to the input conductance according to the usual convention. Indeed it is impossible to match, unless the conditions imposed are abandoned entirely.

The main qualitative conclusions, which are likely to hold generally, are then these; some of them, of course, are familiar. (1) G_{12}, γ_u, G_{45} should be as small as possible. (2) There is a limit beyond which it is undesirable to increase G_v, even if absolute gain is unimportant. (3) The optimum adjustment is definitely not that in which the effective conductance of the signal generator is equal to the input conductance; considerable "overcoupling" is desirable. (4) The effective conductance of the generator can be reduced, below that characteristic of optimum adjustment in respect of N and the A-ratio, without increasing N but not without decreasing the A-ratio.

6. Effect of Lead Inductances in Triode

We now turn to the effect of the lead inductances, assuming that all the quantities other than these are chosen so as to give the optimum adjustment just discussed. It is most unlikely that this assumption will actually be true. For, if we imagine that the amplifier is first adjusted without the inductances, and that they are then introduced, the "tuning" will be upset (e.g., maximum $|A|^2$ will now be appreciably distant from f_0 and the amplifier will be retuned. But, if the effect of the inductances is small, this retuning is sure to affect favourably some of the important characteristics, and is unlikely to affect any of them very unfavourably. Accordingly we may expect that our assumption will exaggerate the effect of the inductances, while leaving the conclusions qualitatively correct.

If all that is required is the effect of a single specified set of inductances, the simplest method is to insert numerical values in (2.2), and to work out Δ and its cofactors numerically. And that may be the quickest way, if facility in the difficult arithmetic of complex quantities has been acquired. But if more than one set has to be considered, it is better to proceed part of the way by algebra. This is quite easy if it is assumed—the assumption is very important—that the inductances are all so small that terms involving their products and squares may be neglected. For then, in developing a determinant, any sub-determinant with a factor L can be equated to zero. One example will be given to show how the work is thereby simplified.

Consider Δ_{54} which, from (2.1), (2.2) is given by

$$x^3\Delta_{54} = -\begin{vmatrix} 1 + j\omega L_1(11)' & j\omega L_1(21) & j\omega L_1(31) & 0 \\ j\omega L_2(12) & 1 + j\omega L_2(22)' & j\omega L_2(32) & -1 \\ (13) & (23) & (33)' & (45) \\ 0 & 0 & -1 & j\omega L_3(45) \end{vmatrix}$$

Expanding by the first row we have

$$x^3\Delta_{54} = -\begin{vmatrix} 1 + j\omega L_2(22)' & j\omega L_2(32) & -1 \\ (23) & (33)' & (45) \\ 0 & -1 & j\omega L_3(45) \end{vmatrix} - j\omega L_1(11)'\begin{vmatrix} 1 & 0 & -1 \\ (23) & (33)' & (45) \\ 0 & -1 & 0 \end{vmatrix}$$

$$+ j\omega L_1(21)\begin{vmatrix} 0 & 0 & -1 \\ (13) & (33)' & (45) \\ 0 & -1 & 0 \end{vmatrix} - j\omega L_1(31)\begin{vmatrix} 0 & 1 & -1 \\ (13) & (23) & (45) \\ 0 & 0 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} (33)' & (45) \\ -1 & j\omega L_3(45) \end{vmatrix} - j\omega L_2(22)'\begin{vmatrix} (33)' & (45) \\ -1 & 0 \end{vmatrix} + j\omega L_2(32)\begin{vmatrix} (23) & (45) \\ 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} (23) & (33)' \\ 0 & -1 \end{vmatrix} - j\omega L_1(11)'\begin{vmatrix} 1 & -1 \\ (23) & (45) \end{vmatrix} - j\omega L_1(21)\begin{vmatrix} (13) & (33)' \\ 0 & -1 \end{vmatrix}$$

$$= -\{(23) + (45)\} + j\omega L_1[-(11)'\{(23) + (45)\} + (21)(13)] - j\omega L_2(22)'(45) - j\omega L_3(45)(33) \quad \ldots \quad (6.1)$$

If we write

$$x^3\Delta_{mn} = D_{mn} + \omega L_1 . d_{mn1} + \omega L_2 . d_{mn2} + \omega L_3 . d_{mn3} \quad .. \quad (6.2),$$

we can find by such methods the values of the terms D and d given in the Appendix. From them we can calculate by straightforward arithmetic, first the Δ_{mn}, and then $N, |A|^2$, G_{04i} given by (2.3)—(2.7).

The result of the calculation is shown in Table II.

At the head of the table are the assumed conductances other than those given in Table I. The inductances, if they are not zero, are taken to be 0.02 microhenry; this is about the value that is found in practice; but, in order to display separately the effect of L_2, a set is included in which $L_2 = 0$, the others being non-zero. Values were actually calculated for 43, 44, 45, 46, 47 Mc/s, partly to provide a check, and partly to provide data for the numerical integration of N, but only some of these are given in the Table. Suffixes indicate to what frequency a quantity refers, m denoting the frequency at which $|A|^2$ is a maximum.

TABLE I

Constants for the triode

Frequency	$f_0 = 45$ Mc/s ; $\delta f = 2$ Mc/s
Capacitances ($\mu\mu F$)	$C_{01} = 3.6$; $C_{12} = 2.5$; $C_{02} = 3.2$
	$C_{13} = 0.03$; $C_{03} = 1.0$; $C_{23} = 1.4$
	$C_u = 1.0$ (valveholder) $+ 2.0$ (coil) $= 3.0$
	$C_v = 1.0$ (valveholder) $+ 2.0$ (coil) $= 3.0$
	$C_{45} = 0.3$ (valveholder) $+ 2.0$ (coil) $= 2.3$
Conductances (*millimho.*)	$g_2 = 7$; $g_3 = 0.07$; $G_{45} = 0.02$; $G_s = 1000/75$
	$\gamma'_u = 0.05$ or 0.1
Electron current	$I_0 = 10$ mA ; $\Gamma^2 = 0.175$

TABLE II

Effect of lead inductances in the triode

$G_u = 0.5$ mmho ; $\gamma_u = 0.05$ mmho ; $G_{12} = 0.05$ mmho ; $G_v = 1.91$ mmho

$G'_u = 0.62$ mmho ; $G'_v = 2.0$ mmho ; $G_{45} = 0.02$ mmho ; $G_s = \dfrac{1000}{75}$

	$L_1 = L_2 = L_3 = 0$;	$L_1 = L_3 = 2 \times 10^{-8}$ H ; $L_2 = 0$	$L_1 = L_2 = L_3 = 2 \times 10^{-8}$ H ;
$(\Delta_{51})_{45}$ (μmho)	$0 + j0$	$27.8 + j22.8$	$27.8 + j22.8$
$(\Delta_{52})_{45}$ (μmho)	$-6980 + j0$	$-6940 + j2.3$	$-7020 - j15.8$
$(\Delta_{53})_{45}$ (μmho)	$+620 + j0$	$+642 + j26.7$	$+636 + j49.7$
$(\Delta_{54})_{45}$ (μmho)	$-6980 + j0$	$-6940 + j2.3$	$-6940 + j6.8$
$(\Delta_{55})_{45}$ (μmho)	$+620 + j0$	$+623 + j26.9$	$+618 + j49.9$
$(\Delta)_{45}$ (mho$^2 \times 10^{-6}$)	$1.38 + j0$	$1.40 + j0.06$	$1.41 + j0.11$
f_m(Mc/s)	45.0	44.9	44.9
$\|A\|^2_m$	170.7	162.6	161.2
$\|A\|^2_{43}$	85.7	85.2	87.8
$\|A\|^2_{44}$	89.4	84.7	81.8
A-ratio	0.502	0.521	0.507
N_m	1.830	1.828	1.820
N	1.879	1.877	1.870
$(G_{04i})_{43}$ (mmho)	0.728	0.720	0.751
$(G_{04i})_{45}$ (mmho)	0.690	0.712	0.728
$(G_{04i})_{47}$ (mmho)	0.725	0.763	0.784

From the values of the Δ_{mn} it appears that the assumption that the effect of the inductances is everywhere small, and that second order terms may be neglected, is everywhere sufficiently fulfilled. None of the effects on N, $|A|^2$, G_{04i} is really serious; the increase in G_{04i} is the largest. All the inductances contribute appreciably to the effects, and that of L_2 is of the same order as those of the others. It appears, therefore, that attempts to decrease L_2 by feeding the grid through several leads in parallel may be misdirected; for the benefit obtained by reducing L_2 by such means is likely to be more than offset by the consequent increase in C_2. (See Section 5).

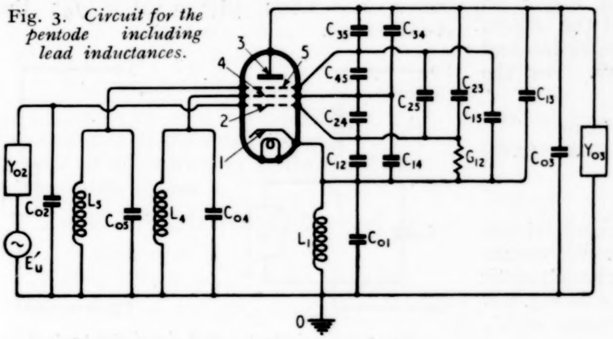
Fig. 3. *Circuit for the pentode including lead inductances.*

7. Common-cathode Pentode without Lead Inductances

We now turn to the pentode, whose diagram is given in Fig. 3. This differs from Fig. 4 of C in two respects. An internal leak G_{12} is shown; this is not likely to be serious in the pentode, but is inserted in order to preserve parallelism with the triode. Again Y_u is replaced by Y_{02} and Y_v by Y_{03}; this is convenient in order that Y_u, Y_v may still mean the admittance in series with the signal e.m.f. and that across which the output is taken. But Y_{02}, Y_{03} do not consist entirely of Y_u, Y_v. Their constitutions are shown in the left-hand parts of Figs. 4 and 5, in which γ_u, C_u, C_v have the same meanings as in Section 2. In the immediately following discussion, in which the inductances are ignored, we have then, by virtue of C(3.12),

$$Y_{02} = Y_u + \gamma_u + j\omega C_u\,;$$
$$Y_v = G_v + j\omega C_v \quad .. \quad .. \quad (7.1)$$

$\varDelta$ is then given by C(5.3), and its elements by C(5.4), if in those elements Y_u, Y_v are replaced by Y_{02}, Y_{03} respectively, and $j\omega C_{12}$ by $G_{12} + j\omega C_{12}$. $\varDelta$ then again reduces to the form (3.1). We shall again write

$$j\omega L_1(11) = 1 + j\omega L_1(11)'\,;$$
$$j\omega L_2 = 1 + j\omega L_2(22)'\,;$$
$$j\omega L_3 = 1 + j\omega L_3(33)' \quad (7.2)$$

The modifications necessary when the inductances are taken into account will be given later.

The sources of noise are the conductors G_u, γ_u, G_{12}, G_v, and the equivalent shot conductor $G_{13\sigma}$. In respect of the last of these we must take partition noise into account and use C(12.2), since $Y_{14} = Y_{04}$ is infinite, and $\rho_{13,14}$ is zero. Hence with the same meaning of γ'_u as in Section 2.

$$N = 1 + \frac{\int[\gamma'_u|\delta_{03,02}|^2 + \{G_v + a\alpha_3 I_0(\alpha_3\Gamma^2 + \alpha_4)\}|\delta_{03,\,03}|^2]\,df}{\int G_u|\delta_{03,\,02}|^2\,df} \quad .. \quad .. \quad .. \quad (7.3)$$

$$\text{with } |\delta_{03,02}|^2 = \alpha^2 g_2{}^2 + \omega^2 C_{23}{}^2\,;\; |\delta_{03,03}|^2 = G'_u{}^2 + B'_u{}^2 \quad .. \quad .. \quad .. \quad .. \quad (7.4)$$

$$|A|^2 = G_u G_s \,.\left|\frac{\delta_{03,02}}{\varDelta}\right|^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (7.5)$$

$$\text{with } |\varDelta|^2 = (G'_u G'_v - B'_u B'_v + \omega^2 C_{23}{}^2)^2 + (G_u{}' B'_v + G'_v B'_u + \alpha_3 g_2 \omega C_{23})^2 \quad .. \quad .. \quad (7.6)$$

where

$$G'_u = G_u + \gamma'_u\,;\; B'_u = B_u + \omega(C_u + C_2)$$
$$G'_v = G_v + \alpha_3 g_3\,;\; B'_v = B_v + \omega(C_v + C_3) \quad .. \quad .. \quad .. \quad .. \quad (7.7)$$

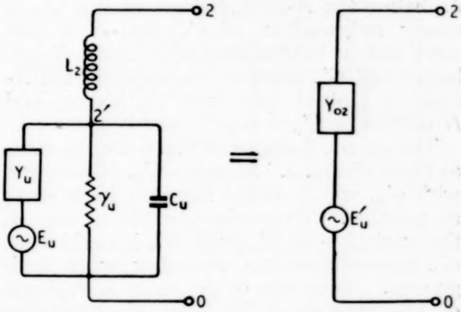
Fig. 4. *Equivalent input circuit for pentode.*

We make the first assumption of Section 4 about B'_u, B'_v, and the third about the conductances. We make the second assumption concerning B'_u, so that

$$B'_u = y\omega_0(C_u + C_2) \quad .. \quad .. \quad (7.8)$$

But, in order that the frequency-dependent terms in the denominator of $|A|^2$ may vanish at f_0, we must adjust B_v so that the second bracket of 7.6) vanishes. The resulting value of B'_v is

$$B'_v = y\omega_0(C_v + C_3) - \frac{\omega_0}{\omega} \cdot \frac{\alpha_3 g_2}{G'_u} \cdot \omega_0 C_{23} \qquad (7{\cdot}9)$$

The terms $\omega^2 C_{23}{}^2$ remain, but C_{23} is so small, owing to the presence of the screen, that this term is completely negligible and will be ignored in what follows. For the input conductance we have

$$G_{02i} = \Delta/\Delta_{22} = G'_u + \text{(a term that is always small)} \quad .. \quad (7.10)$$

8. Adjustment of Pentode

We now discuss the adjustment of the pentode on the lines of Section 6. We assume the values given in Table III as characteristic of a modern r.f. pentode, and plot in Fig. 6 N and the A-ratio with selected values of G'_v. As before the A-ratio plotted against G'_u is nearly independent of γ'_u, and N is still more nearly independent of G'_v, since $G_{13\sigma}$ is larger and G'_v smaller. Accordingly the A-ratio is plotted only with $\gamma'_u = 0.05$, and N only with $G'_v = 0.4$.

The general features of Fig. 6 are the same as those of Fig. 2. At given G'_u, N increases with γ'_u, which should therefore be as small as possible. When the A-ratio is near $\frac{1}{2}$, the variation of G'_u with G'_v is rapid when G'_r is less than 1.0 and slow when it is greater. Since one of the main advantages of the pentode is its large absolute gain, it may be unwise to choose G'_v larger than is absolutely necessary; so we shall take $G'_v = 0.4$ for further discussion. If γ'_u is as great as 0.05, it is then possible to choose G'_u so that N is very near its minimum, but not if γ'_u is as small as 0.025. Since we shall choose $\gamma'_u = 0.05$ for further discussion, we shall choose $G'_u = 0.4$; G_{02i} is practically the same as G'_u, so again there is great overcoupling. With these values $N = 1.80$; $|A|^2 = 10^4$ at f_0; the A-ratio 0.55.

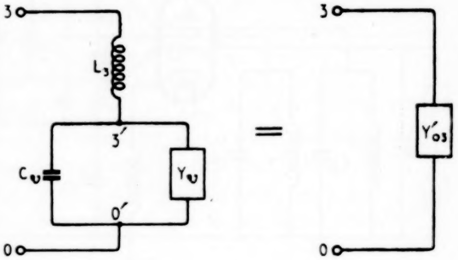

Fig. 5. *Equivalent output circuit for pentode.*

9. Effect of Lead Inductances in Pentode

In order to apply the method of section 6 to the lead inductances, we must take into account L_2, L_3 in Figs. 4 and 5. One way of doing this is to introduce new terminals at 2′, 3′ and to develop the resulting 49-element determinant. This is the most certain method, and the best for those skilled in handling determinants. Another way not available in Section 6, is to transform the left-hand parts of Figs. 4 and 5 into the right-hand parts of Thévenin's theorem; it has the advantage of introducing the necessary modifications in a form in which their physical significance is evident; it will be adopted here.

TABLE III

Constants of the Pentode

Frequency	$f_0 = 45$ Mc/s	$\Delta f = 2$ Mc/s	
Capacitances ($\mu\mu F$)	$C_{01} = 0.3$	$C_{12} = 5.0$	$C_{24} = 2.6$
	$C_{02} = 0.4$	$C_{13} = 0.002$	$C_{25} = 0.8$
	$C_{03} = 1.4$	$C_{14} = 0.05$	$C_{34} = 0.1$
	$C_{04} = 0.3$	$C_{15} = 0.3$	$C_{35} = 1.5$
	$C_{05} = 0.5$	$C_{23} = 0.005$	$C_{45} = 2.0$
	$C_u = C_v = 1.0$ (valveholder) $+ 2.0$ (coil) $= 3.0$		
Conductances (*mmho*)	$g_2 = 9.4$; $g_3 = 0.004$; $g_4 = 0.20$; $g_5 = 0.20$		
	$G_s = 1000/75$; $G_{12} = 0$; $\gamma_u = \gamma'_u = 0.025$ or 0.050 or 0.075.		
Electron current	$I_0 = 12.5$ mA; $\Gamma^2 = 0.175$; $\alpha_3 = 0.8$; $\alpha_4 = 0.2$.		

If Y_{02}, Y_{03} are still given by (7.1), we have

$$Y'_{02} = \frac{Y_{02}}{1 + j\omega L_2 \,.\, Y_{02}} ;$$

$$Y'_{03} = \frac{Y_{03}}{1 + j\omega L_3 \,.\, Y_{03}} \quad .. \quad (9.1)$$

$$E'_u = E_u \,.\, Y_u/Y_{02} ;$$
$$E'_v = E_v \,.\, Y_v/Y_{03} \quad .. \quad .. \quad (9.2)$$

Y'_{02}, Y'_{03} must be used in place of Y_{02}, Y_{03} in the elements of the determinant. To the first order of the terms in L their real and imaginary parts are

$$G'_{02} = (G_u + \gamma_u)\{1 + 2\omega L_2(B_u + \omega C_u)\};$$
$$jB'_{02} = j(B_u + \omega C_u) + j\omega L_2\{(B_u + \omega C_u)^2 - (G_u + \gamma_u)^2\},$$
$$G'_{03} = G_v\{1 + 2\omega L_3(B_v + \omega C_v)\} ;$$
$$jB'_{03} = j(B_v + \omega C_v) + j\omega L_3\{(B_u + \omega C_v)^2 - G_v^2\} \,.. \quad (9.3)$$

Also, in virtue of C(3.15), when considering gains, we must replace E_uY_u, E_vY_v, by $\chi_uE_uY_u$, $\chi_vE_vY_v$ respectively, where

$$\chi_u = 1/(1 + j\omega L_2 \,.\, Y_{02}) ;$$
$$|\chi_u|^2 \approx 1 + 2\omega L_2(B_u + \omega C_u) \quad (9.4)$$
$$\chi_v = 1/(1 + j\omega L_3 \,.\, Y_{03}) ;$$
$$|\chi_v|^2 \approx 1 + 2\omega L_3(B_v + \omega C_v)$$

We thus have, using C(8.11) for the partition noise,

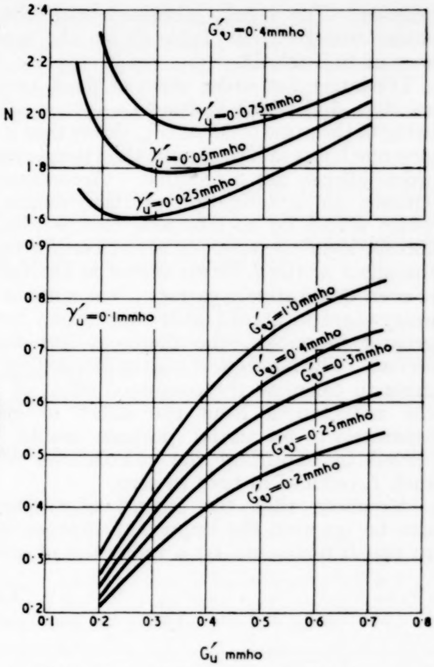

Fig. 6. *Variation of N and A-ratio with generator coupling for a pentode with no lead inductances.*

$$N = 1 + \frac{\int[\gamma_u|\delta_{03,02}|^2|\chi_u|^2 + G_{12}|\delta_{0312}|^2 + G_{03}|\delta_{03,03}|^2]|\chi_v|^2\,df}{\int G_u|\delta_{03,02}|^2|\chi_u|^2|\chi_v|^2\,df} + $$
$$+ \frac{\int[aI_0\{\Gamma^2|\alpha_3\delta_{03,13} + \alpha_4\delta_{03,14}|^2 + \alpha_3\alpha_4|\,\delta_{03,13} - \delta_{03,14}|\}^2]|\chi_v|^2\,df}{\int G_u|\delta_{03,02}|^2|\chi_u|^2|\chi_v|^2\,df} \quad .. \quad .. \quad (9.5)$$

$$|A|^2 = G_uG_s\left|\frac{\delta_{03,02}}{\Delta}\right|^2 |\,\chi_u|^2|\chi_v|^2 \quad .. \quad (9.6)$$

where

$$\Delta = (31)\Delta_{31} + (32)\Delta_{32} + (33)\Delta_{33} + (34)\Delta_{34} + (35)\Delta_{35} \quad .. \quad (9.7)$$

and the δ terms refer to the 25-element determinant of C(5.3).

When we seek the input conductance, we must remember that Y_{02i} of C(3.17), which is Δ/Δ_{22}, is the admittance looking into the terminals (0,2), while we require the conductance (say G'_{02i}) looking into the terminals (0,2′). Since Y_{02i} is made up of an external admittance Y_{02} and an internal admittance $(Y_{02i} - Y_{02})$ in parallel between 0 and 2, we have

$$Y'_{02i} = Y_u + \gamma_u + j\omega C_u + \frac{Y_{02i} - Y_{02}}{1 + j\omega L_2\,(Y_{02i} - Y_{02})} \quad .. \quad (9.8)$$

or

$$G'_{02i} \approx G_u + \gamma_u + (G_{02i} - G_{02})\{1 + 2\omega L_2(B_{02i} - B_{02})\} \quad .. \quad (9.9)$$

The cofactors of Δ, retaining only the first order terms in L, can be found by the method of Section 6. They are given in the Appendix with the notation

$$\Delta_{mn} = D_{mn} + \omega L_1 \,.\, d_{mn1} + \omega L_2 \,.\, d_{mn2} + \omega L_3 \,.\, d_{mn3} + \omega L_4 \,.\, d_{mn4} + \omega L_5 \,.\, d_{mn5} \quad .. \quad (9.10)$$

But, since the purpose of the screen is to make g_3, C_{13}, C_{23} small, these are ignored when they occur in the d terms and are multiplied by an L; they are not ignored in the D terms.

Measurements indicate that appropriate values of the L terms are $L_1 = 0.04$, $L_2 = L_3 = L_4 = 0.02$, $L_5 = 0.01$ microhenry. The larger value of L_1 is partly due to the inherent inductance of the cylindrical cathode. The results obtained with these values are given in Table IV in the same form as in Table II.

The larger first-order effect of the L terms on Δ_{31}, Δ_{33}, which is due to $\omega L_{1} \, . \, {}_{g2} \, . \, \omega C_{12}$ being of the same order as G'_{u}, shows that it is now much less safe to assume that the second-order effects are negligible. Accordingly, without an investigation of these effects—which would be so laborious that we have shrunk from the task—it is not certain that the effect of the L terms shown in the table is even qualitatively correct ; but since it is everywhere in accord with expectation from general principles, the doubt is not very serious. The large effect also makes it illegitimate to calculate the combined effect of all the inductances from the effect of each separately ; no useful purpose would be served by repeating the calculations with each L reduced in turn to zero.

Assuming that the second-order effects can be ignored, the important changes due to the L terms are (1) a reduction of some 2 db in the signal gain, (2) some increase in the A-ratio, with marked dis-symmetry in the response curve, (3) an increase of some 0.5 db in N, (4) an increase of some 40 per cent in the input conductance.

10. Comparison of Triode and Pentode

A comparison of Tables II and IV with the inductances present shows that the pentode has the advantages of giving a much larger signal gain and a considerably smaller input conductance. The first of these is due mainly to a smaller G_{v}, the second to a smaller G_{u}. The reason why it is possible to use a smaller G_{u} and G_{v} in the pentode lies in the absence of the third tuned circuit between grid and anode ; for the presence of this circuit tends to narrow the response curve and therefore to require more damping, for a given A-ratio, in the input and output circuits. On the other hand the pentode has an appreciably greater noise factor. This is, of course, due primarily to the presence of partition noise : indeed it might be expected that the increase

TABLE IV

Effect of lead inductances in the pentode

$G_{u} = 0.35$ mmho ; $\gamma_{u} = 0.05$ mmho ; $G_{12} = 0$; $G_{v} = 0.397$ mmho ; $G'_{u} = G'_{v} = 0.400$ mmho.

	$L_1 = L_2 = L_3 = L_4 = L_5 = 0$	$L_1 = 4 \times 10^{-8}$ H ; $L_5 = 1 \times 10^{-8}$ H ; $L_2 = L_3 = L_4 = 2 \times 10^{-8}$ H.
$(\Delta_{31})_{45}$ (μmho)	$0 + j0$	$+ 119.8 + j\,35.4$
$(\Delta_{32})_{45}$,,	$- 7500.0 + j1.4$	$- 7397.4 + j\,1.0$
$(\Delta_{33})_{45}$,,	$+ 400.0 + j0$	$+ 518.6 + j104.2$
$(\Delta_{34})_{45}$,,	$+ 0 + j0$	$+ 31.1 - j\,0.4$
$(\Delta_{35})_{45}$,,	$+ 0 + j0$	$+ 4.3 - j\,0.2$
$(\Delta)_{45}$ (mho$^2 \times 10^{-6}$)	$0.161 + j0$	$.209 + j0.03$
f_m	45 Mc/s	44.6 Mc/s
$\lvert A\rvert^2_{m}$	1.009×10^4	0.592×10^4
$\lvert A\rvert^2_{43}$	0.577×10^4	0.439×10^4
$\lvert A\rvert^2_{47}$	0.557×10^4	0.364×10^4
A-ratio	0.551	0.614
N_m	1.699	1.894
N	1.801	2.003
$(G'_{02i})_{43}$ (mmho)	0.398	0.545
$(G'_{02i})_{45}$,,	0.400	0.561
$(G'_{02i})_{47}$,,	0.408	0.587

due to this cause would be greater. It may be surprising that, in the absence of the inductances, the noise factor is actually greater in the triode. This is due to the choice of a larger γ'_u in the triode, arising from the assumed presence of a leak G_{12}; if the same γ'_u were assumed in both cases, N would always be less in the triode than in the pentode. But that is not the whole story. $G_{13\sigma}$ always appears in N multiplied by a factor that increases with G_u, while γ'_u is multiplied by a factor that decreases when G_u increases. Accordingly, though increase of $G_{13\sigma}$ increases N at constant G_u, it displaces the minimum of N to smaller G_u, and is therefore, if the minimum of N can be used, partly self-compensating. The possibility of approaching minimum N nearly both in triode and pentode, although G_u is much less for the latter, is again due to the smaller damping required in the pentode.

11. Acknowledgment

In conclusion, the authors desire to tender their acknowledgments to the M.O. Valve Co. Ltd., on whose behalf the work described in this paper was carried out.

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[1] N. R. Campbell, V. J. Francis and E. G. James, *Wireless Engineer*, March and April 1946, Vol. 23, pp. 74 and 116.

[2] N. R. Campbell, V. J. Francis and E. G. James, *Wireless Engineer*, May 1948, Vol. 25, p. 148.

APPENDIX

For Triode $\qquad x^3\Delta_{mn} = D_{mn} + \omega L_1 \,.\, d_{mn1} + \omega L_2\, d_{mn2} + \omega L_3 \,.\, d_{mn3} \qquad g_2 = g''_2 + g_3 + G_{12}$

$$D_{51} = d_{512} = d_{513} = 0$$

$$d_{511} = +j[(12)\{(23)+(45)\} - (13)\{(22)'+(44)'\} = g'_2\omega C_{12} - G'_u\omega C_{13} - G_{12}B'_{45} - (g_2+g_3)B'_u + j\{-g'_2G_{12} + G'_u(g_2+g_3) - \omega C_{12}B'_{45} - \omega C_{13}B'_u\}$$

$$D_{52} = -\{(23+45)\} = -g_2 + jB'_{45}$$

$$d_{521} = -j[(11)'\{(23)+(45)\} - (21)(13)] = g'_2\omega C_1 - (g_2+g_3)\omega C_{12} - (g_2+G_{12})\omega C_{13} - g''_2B'_{45} + j\{-g'_2g''_2 + (g_2+G_{12})(g_2+g_3) - \omega^2C_{12}C_{13} - \omega C_1B'_{45}\}$$

$$d_{522} = -j(44)'(23) = g_2B'_4 - (G'_u - G_{12})\omega C_{23} - g_2\omega C_2 + j\{-g_2(G'_u - G_{12}) + \omega^2C_2C_{23} - \omega C_{23}B'_u\}$$

$$d_{523} = -j(33)'(45) = -g_3(B'_{45} - \omega C_{23}) - G_{45}\omega C_3 + j\{g_3G_{45} - \omega C_3(B'_{45} - \omega C_{23})\}$$

$$D_{53} = (22)' + (44)' = G'_u + jB'_u$$

$$d_{531} = +j[(11)'\{(22)'+(44)'\} - (21)(12)] = -G'_u\omega C_1 + (g_2 + 2G_{12})\omega C_{12} - g''_2B'_u + j\{g''_2G'_u - (g_2+G_{12})G_{12} + \omega^2C_{12}^2 - \omega C_1B'_u\}$$

$$d_{532} = +j(22)'(44)' = -(G'_u - 2G_{12})\omega C_2 - G_{12}B'_u + j\{G_{12}(G'_u - G_{12}) - \omega C_2B'_u + \omega^2C_2^2\}$$

$$d_{533} = +j(32)(45) = -G_{45}\omega C_{23} - j\omega C_{23}(B'_{45} - \omega C_{23})$$

$$D_{54} = D_{52}\,;\ d_{541} = d_{521}\,;$$

$$d_{543} = d_{523}\,;\ d_{542} = -j(22)'(45) = -G_{45}\omega C_2 - G_{12}(B'_{45} - \omega C_{23}) + j\{G_{12}G_{45} - \omega C_2 (\mathrm{B}'_{45} - \omega C_{23})\}$$

$$D_{55} = D_{53}\,;\ d_{551} = d_{531}\,;\ d_{552} = d_{532}$$

$$d_{553} = +j[(33)'\{(22)'+(44)'\} - (23)(32)] = -G'_u\omega C_3 - g_2\omega C_{23} - g_3B'_u + j\{g_3G'_u + \omega^2C_{23}^2 - \omega C_3B'_u\}$$

$$D_{44} = (33)' + (55)' = G'_v + jB'_v$$

$$d_{441} = +j[(11)'\{(33)'+(55)'\} - (31)(13)] = -G'_v \,.\, \omega C_1 + (g_2 + 2g_3)\omega C_{13} - g''_2B'_v + j\{g''_2G'_v - g_3(g_2+g_3) + \omega^2C_{13}^2 - \omega C_1B'_v\}$$

$$d_{442} = +j[(22)'\{(33)'+(55)'\} - (32)(23)] = -G'_v \,.\, \omega C_2 - g_2\omega C_{23} - G_{12}B'_v + j\{G'_vG_{12} + \omega^2C_{23}^2 - \omega C_2B'_v\}$$

$$d_{443} = +j(33)'(55)' = -g_3B'_v - (G'_v - 2g_3)\omega C_3 + j\{g_3(G'_v - g_3) + \omega^2C_3^2 - \omega C_3B'_v\}$$

For Pentode $\qquad x^3\Delta_{mn} = D_{mn} + \omega L_1d_{mn1} + \omega L_2 \,.\, d_{mn2} + \omega L_3 \,.\, d_{mn3} + \omega L_4 \,.\, d_{mn4} + \omega L_5 \,.\, d_{mn5}$

$$D_{31} = d_{312} = d_{313} = d_{314} = d_{315} = 0$$

$$d_{311} = +j[(12)(23) - (13)(22)] = \alpha_3g_2 \,.\, \omega C_{12} + \alpha_3g_1 \,.\, B'_u + j\{-\alpha_3g_1G'_u - \alpha_3g_2G_{12}\}$$

$$D_{32} = -(23) = -\alpha_3g_2 + j\omega C_{23}$$

$$d_{321} = -j[(11)'(23) - (21)(13)] = \alpha_3g_2 \,.\, \omega C_1 + \alpha_3g_1 \,.\, \omega C_{12} + j\{-G_{12}\alpha_3(g_1+g_2)\}$$

$$d_{322} = d_{323} = 0$$

$$d_{324} = -j[(44)'(23) - (24)(43)] = \alpha_3g_2 \,.\, \omega C_4 + \alpha_3g_4 \,.\, \omega C_{24} + \alpha_4g_2 \,.\, \omega C_{34} + j\{-\omega^2C_{24}C_{34}\}$$

$$d_{325} = -j[(55)'(23) - (25)(53)] = \alpha_3g_2 \,.\, \omega C_5 + \alpha_3g_5 \,.\, \omega C_{25} + j\{-\omega^2C_{25}C_{35}\}$$

$$D_{33} = (22) - \omega L_2 \,.\, d_{332} = G'_u + jB'_u$$

$$d_{331} \quad + j[(11)'(22) - (21)(12)] = - G'_u \,.\, \omega C_1 + (g_2 + 2G_{12})\omega C_{12} + (g_1 - G_{12})B'_u + j\{(- g_1 + G_{12})G'_u - (g_2 + G_{12})G_{12} + \omega^2 C_{12}{}^2 - \omega C_1 \;\; B'_u\}$$

$$d_{332} = 2(G'_u - G_{12})(B'_u - \omega C_2) + j\{- (G'_u - G_{12})^2 + (B'_u - \omega C_2)^2\}$$

$$d_{333} = 0$$

$$d_{334} = + j[(44)'(22) - (24)(42)] = G'_u \,.\, \omega C_4 - \alpha_4 g_2 \,.\, \omega C_{24} - \alpha_4 g_4 B'_u + j\{\alpha_4 g_4 G'_u + \omega^2 C_{24}{}^2 - B'_u \,.\, \omega C_4\}$$

$$d_{335} = + j[(55)'(22) - (25)(52)] = - G'_u \,.\, \omega C_5 + j\{\omega^2 C_{25}{}^2 - \omega C_5 \,.\, B'_u\}$$

$$D_{34} = d_{341} = d_{342} = d_{343} = d_{345} = 0$$

$$d_{344} = - j[(22)(43) - (23)(42)] = - G'_u \,.\, \omega C_{34} + \alpha_3 g_2 \,.\, \omega C_{34} + \alpha_3 g_4 \,.\, B'_u - j\{\alpha_3 g_4 G'_u + \omega C_{34} \,.\, B'_u\}$$

$$D_{35} = d_{351} = d_{352} = d_{353} = d_{354} = 0$$

$$d_{355} = - j[(22)(53) - (23)(52)] = - G'_u \,.\, \omega C_{35} + \alpha_3 g_2 \,.\, \omega C_{35} + \alpha_3 g_5 \,.\, B'_u - j\{a_3 g_5 G'_u + \omega C_{35} \,.\, B'_u\}$$

$$D_{22} = (33) - \omega L_3 \,.\, d_{223} = \alpha_3 g_3 + G_v + jB'_v = G'_v + jB'_v$$

$$d_{221} \quad + j[(11)'(33) - (31)(13)] = - G_v \,.\, \omega C_1 + (g_1 - G_{12})B'_v + j\{(- g_1 + G_{12})G_v - \omega C_1 \,.\, B'_v\}$$

$$d_{222} = 0$$

$$d_{223} = \quad = 2G_v(B'_v - \omega C_3) + j\{- G_v{}^2 + (B'_v - \omega C_3)^2\}$$

$$d_{224} = + j[(44)'(33) - (34)(43)] = - G_v \,.\, \omega C_4 - \alpha_3 g_4 \,.\, \omega C_{34} - \alpha_4 g_4 B'_v + j\{\alpha_4 g_4 G_v - \omega C_4 \,.\, B'_v + \omega^2 C_{34}{}^2\}$$

$$d_{225} = + j[(55)'(33) - (35)(53)] = - G_v \,.\, \omega C_5 - \alpha_3 g_5 \,.\, \omega C_{35} + j\{- \omega C_5 B'_v + \omega^2 C_{35}{}^2\}$$

DESIGN OF SHUNT EQUALIZERS

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SUMMARY.—A method of designing shunt equalizers with precision is described and examples of results achieved by its use are given.

1. Introduction

ALTHOUGH the shunt equalizer has had wide application in telecommunication engineering, information on the theory of its performance and design, particularly with regard to its use as a line equalizer as distinct from its role as a corrector connected in the control-grid circuit of a valve amplifier, is not readily obtainable. Equalizers of this type are commonly regarded as being unamenable to calculation (indeed, in an article on equalization which appeared recently, it was explicitly stated that their design was simply a matter of guesswork), and as capable of yielding only mediocre results, a viewpoint in the establishment of which their simplicity and cheapness may have played some part.

It is the purpose of this article to describe a method for designing such equalizers with precision, and to show that by their use good equalization can be obtained. A typical shunt equalizer circuit is given in Fig. 1.

In a previous article[1] it has been shown how the performance of any given equalizer can be predicted and the results are, of course, applicable to shunt equalizers. It remains, therefore, to find means by which the values of the components of the latter can be calculated to provide a given amount of equalization. For ladder-type equalizers a simplification can be effected by regarding the shunt arm as a frequency-variable potentiometer which caters for the main portion of the frequency distortion to be corrected, the series arm being built out so as to adjust the equalizer input impedance to equalize variations caused by reflection effects. With the shunt equalizer the problem of design is complicated by the absence of a series arm, which means that the potentiometer and reflection effects are mutually dependent, and that the input impedance of the equalizer is subject to wide variations with frequency which result in marked variations in reflection losses.

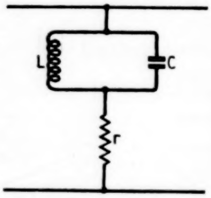

Fig. 1. *Typical shunt equalizer.*

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The theoretical treatment which follows is based on the following assumptions:

(a) Insertion loss is a suitable criterion of equalization.

(b) Post-equalization is employed.

(c) Transmitters and receivers, or their equivalents in the form of thermionic valve amplifiers, are ideal.

(d) The circuit to be equalized is capable of passing zero-frequency currents. This reservation is made mainly to preserve consistency in the mathematical argument, and is normally of little importance in practice. In most instances where series capacitors are encountered, the design can be handled as if they were absent, the only effect being a cut-off at low frequencies which can readily be estimated in predicting the performance of the finished equalizer.

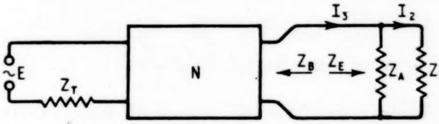

Fig. 2. *Basic elements of a circuit incorporating a shunt equalizer.*

2. Design Formulae

In Fig. 2 is sketched the basic elements of the equalized circuit, N being a network equivalent to the circuit to be equalized, and Z_T, Z_R the impedances of the transmitter and receiver respectively.

The objective in equalization may broadly be described by reference to Fig. 3, in which AB represents the insertion loss characteristic of N for terminating impedances Z_T, Z_R, and f_c is the upper limit of the frequency range over which equalization is required. Ideally, the insertion-loss characteristic of the equalized circuit would be CD, which means, roughly speaking, that at any frequency f the equalizer must introduce a loss x which when added to w gives the constant y. In practice a result such as EF will be obtained.

Referring to Figs. 2 and 3, let

E = sinusoidal transmitter e.m.f.

Z_T = transmitter impedance.

$Z_R = R_R + jX_R = P\underline{/\phi}$ = receiver impedance.

Z_B = impedance looking back into N with the sending terminals of the latter terminated in Z_R.

Z_E = impedance presented by Z_A and Z_R in parallel.

$Z_A = R_A + jX_A$ = impedance of equalizer.

I_1 = current to Z_R when directly connected to Z_T without any intervening circuit.

I_2 = current to Z_R.

I_3 = current leaving N.

$\frac{I_3}{I_2} = K$, where K is, in general, complex.

$1 + \frac{Z_R}{Z_B} = z_m = n\underline{/\theta} = r_m + jx_m.$

A_0 = overall insertion loss of the equalized circuit between impedances Z_T, Z_R.

$M = \text{antilog}_{10}\left(\frac{x}{10}\right)$

Then, as has been shown elsewhere[1]

$$A_0 = 20 \log_{10} \left|\frac{I_1}{I_2}\right|$$

$$= w + 20 \log_{10} \left| K \times \frac{Z_B + Z_E}{Z_B + Z_R} \right| \quad (1)$$

$$\text{Now } K = \frac{Z_A + Z_R}{Z_A} = 1 + \frac{Z_R}{Z_A} \quad .. \quad (2)$$

$$\text{and } Z_E = \frac{Z_A Z_R}{Z_A + Z_R} = \frac{Z_R}{K} \quad \ldots \quad .. \quad (3)$$

Substituting (2) and (3) in (1) we obtain

$$A_0 = w + 20 \log_{10} \left| \text{K}. \frac{Z_B + \frac{Z_R}{K}}{Z_B + Z_R} \right|$$

$$= w + 20 \log_{10} \left| \text{K}. \frac{1 + \frac{Z_R}{Z_B K}}{1 + \frac{Z_R}{Z_B}} \right|$$

$$= w + 20 \log_{10} \left| \frac{K + \frac{Z_R}{Z_B}}{1 + \frac{Z_R}{Z_B}} \right|$$

$$= w + 20 \log_{10} \left| \frac{1 + \frac{Z_R}{Z_B} + \frac{Z_R}{Z_A}}{1 + \frac{Z_R}{Z_B}} \right|$$

$$= w + 20 \log_{10} \left| \frac{Z_A z_m + Z_R}{Z_A z_m} \right| \quad .. \quad (4)$$

For design purposes we put $A_0 = y$ and from (4) obtain

$$x = y - w = 20 \log_{10} \left| \frac{Z_A z_m + Z_R}{Z_A z_m} \right|$$

$$\text{or antilog}_{10}\left(\frac{x}{10}\right) = M = \left| \frac{Z_A z_m + Z_R}{Z_A z_m} \right|^2 \quad (5)$$

Hence

$$M = \left| \frac{(R_A + jX_A)(r_m + jx_m) + (R_B + jX_B)}{Z_A z_m} \right|^2$$

$$= \left| \frac{(R_A r_m - X_A x_m + R_B) + j(r_m X_A + R_A x_m + X_B)}{Z_A z_m} \right|^2$$

$$= \frac{n^2 X_A^2 + 2X_A(r_m X_B - R_B x_m) + n^2 R_A^2 + 2R_A(r_m R_B + x_m X_B) + P^2}{n^2(R_A^2 + X_A^2)}$$

or

$$X_A^2(M-1)n^2 - 2X_A(r_m X_B - R_B x_m) + R_A^2(M-1)n^2 - 2R_A(r_m R_B + x_m X_B) - P^2 = 0$$

whence $X_A =$ .. .. (6)

$$\frac{(r_m X_B - R_B x_m) \pm \sqrt{(r_m X_B - R_B x_m)^2 - n^2(M-1)\{n^2 R_A^2(M-1) - 2R_A(r_m R_B + x_m X_B) - P^2\}}}{n^2(M-1)}$$

and $R_A =$ .. .. (7)

$$\frac{(r_m R_B + x_m X_B) \pm \sqrt{(r_m R_B + x_m X_B)^2 - n^2(M-1)\{n^2 X_A^2(M-1) - 2X_A(r_m X_B - R_B x_m) - P^2\}}}{n^2(M-1)}$$

.. .. .. (8)

Equations (7) and (8) both contain two unknowns. Providing that R_A is independent of frequency—and this is almost invariably true—its value can be evaluated from (8) by considering the special case of very low frequencies when X_A vanishes. Under this condition we also have, incidentally, $x_m = 0$, and $n = r_m$, and (8) becomes

$$R_A = \frac{r_m R_B \pm \sqrt{r_m^2 R_B^2 + r_m^2(M-1)P^2}}{r_m^2(M-1)}$$

$$= \frac{R_B \pm \sqrt{R_B^2 + (M-1)P^2}}{r_m(M-1)} \quad .. \quad (9)$$

R_A having been determined, its substitution in (7) permits X_A to be found.

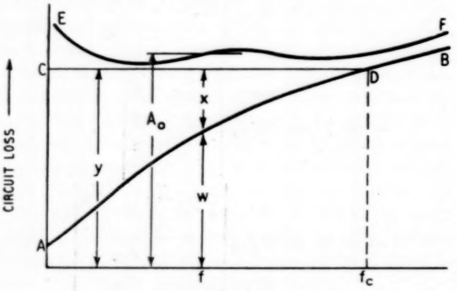

Fig. 3. *Ideal and practical frequency response curves for an equalized network.*

In practice Z_B is usually purely resistive and may be designated $R\angle 0$. Putting R for Z_B in (7) and (9) we obtain

$$X_A = \frac{-R x_m \pm \sqrt{R^2 x_m^2 - n^2(M-1)\{n^2 R_A^2(M-1) - 2R_A R r_m - R^2\}}}{n^2(M-1)} \quad .. \quad (10)$$

and $R_A = R\left\{\frac{1 \pm \sqrt{M}}{r_m(M-1)}\right\}$ .. .. (11)†

In addition (4) becomes

$$A_0 = w + 20\log_{10}\left|\frac{Z_A z_m + R}{Z_A z_m}\right| \quad .. \quad (12)$$

With regard to the apparent uncertainty created by the existence of two possible solutions for (10) and (11) it will be found that, for practical purposes, there is no difficulty. Since M cannot be less than unity it is clear that in (11) the positive value of the quantity under the root sign should always be taken. Since x_m is usually positive, similar considerations normally apply to (10). Where x_m is negative any ambiguity which may arise initially will be cleared up as the design proceeds.

It will sometimes be found that the value of X_A obtained from (10) is imaginary. This is due to the fact that the value of R_A found from (11), while being suitable at very low frequencies, is too high for higher frequencies. The remedy is to choose a larger value of y (i.e., virtually to increase f_c) and recommence the design.

At first sight the collection of the circuit data required for the evaluation of R_A and X_A may appear to present a somewhat formidable task, but in fact this is not so. In the great majority of instances the circuit to be equalized is both uniform in composition and electrically "long." It has been shown that under these conditions, Z_0, the characteristic impedance of the circuit, can be used in place of Z_B (except in (11), but here merely a loop resistance is required), so that R_m and X_m then become standard data for the particular type of circuit involved. The quantity w can similarly be so regarded, although for small-gauge cables actual measurements of w over the desired frequency band are desirable if high accuracy is required.

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†An alternative method of finding R_A is given in the Appendix.

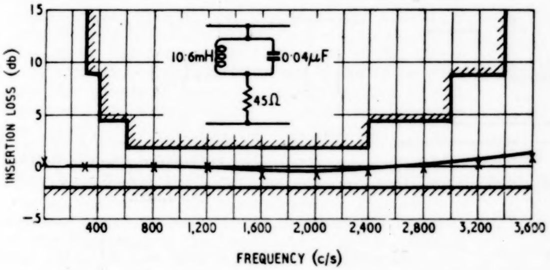


Fig. 4. *Response of 19.2 miles of 40-lb cable equalized : ——— measured ;* x — x — x *calculated.*

3. Design Procedure

From the foregoing, the following method of design may be deduced.

(1) From the value of M corresponding to zero frequency R_A is calculated by means of (11) or (18), the resistive component of the equalizer being thereby determined.

(2) By using (10) X_A can then be computed for several frequencies and from these results the corresponding values of the reactive components can be found. It is improbable that the latter will be exactly the same for every frequency chosen and an average value must be struck.

(3) The overall performance of the equalizer so designed can then be checked by means of (12) and the general trend of any trimming adjustments which may be necessary will be evident by inspection. In making alterations, the guiding principle is that R_A will be determining at low frequencies and X_A at frequencies towards the upper portion of the frequency band to be covered.

4. Practical Examples

As an illustration of the foregoing, consider the design of the familiar type of equalizer shown in Fig. 1. Here $R_A = r$, so that r can be found directly from (11) or (18).

L and C will resonate at some frequency f_t. Ideally, since the transmission loss caused by the equalizer should be zero at f_c, f_t should be made to coincide with the latter.

Writing $\omega_c = 2\pi f_c$ we have

$$LC = \frac{1}{\omega_c^2} \quad \cdots \quad \cdots \quad \cdots \quad (13)$$

Also

$$X_A = \frac{\omega L}{1 - \omega^2 LC} = \frac{\omega L}{1 - \dfrac{\omega^2}{\omega_c^2}} = \frac{\omega L}{\eta} \quad (14)$$

where $\eta = 1 - \dfrac{\omega^2}{\omega_c^2}$.

Hence

$$L = \frac{\eta X_A}{\omega} \quad \cdots \quad \cdots \quad \cdots \quad (15)$$

from which L can be found for $f = \dfrac{\omega}{2\pi}$ corresponding to the value of X_A obtained from (10). C follows from (13).

It will often be found that the equalization can be improved by arranging f_t to be considerably in excess of f_c, the penalty being, of course, some increase in the overall transmission loss, although this is normally of no importance. For these conditions $\omega_t = 2\pi f_t$ will replace ω_c in (13), (14), and (15).

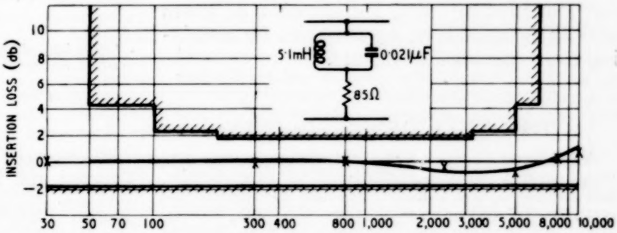


Fig. 5. *Response of 6.33 miles of 20-lb cable equalized as a music circuit ; ——— measured ;* x — x — x *calculated.*

It is to be noted that the resistance of the inductor L should be included in R_A; but can, for modern inductors, be neglected as far as the LC combination is concerned.

In Figs. 4 and 5 are shown frequency response curves for experimental cable circuits equalized by means of shunt-type equalizers (the circuits of which are shown inset) the insertion loss at 800 c/s being taken as zero datum line. The shaded lines show the C.C.I.F. quality limits for international audio (four-wire) and music circuits respectively.

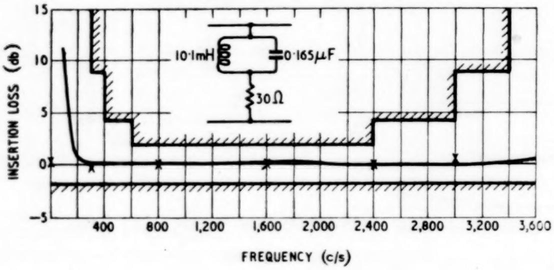


Fig. 6. *Response of* 13.25 *miles of* 10-*lb cable repeatered and equalized.* ——— *measured,* x — x — x *calculated.*

The curve of Fig. 6 was obtained for a working four-wire repeatered telephone trunk, the cut-off at the lower extremity of the frequency band being, of course, due to the repeaters.

In these examples R was $600\angle 0$ ohms, and the designs were worked out by means of the method outlined above. It will be seen that there is close agreement between the actual and predicted performance. The size of cable employed is given in terms of the weight of one mile of single conductor.

4. Acknowledgment

The Author wishes to express his thanks to the Engineer-in-Chief, Department of Posts and Telegraphs Eire, for facilities for obtaining the experimental results.

APPENDIX

Alternative Method of Determining R_A

The circuit conditions obtaining when $f = 0$ are as shown in Fig. 7, R_L being the loop resistance of the circuit to be equalized.

$$I_3 = \frac{E}{R+R_L+\dfrac{R_A R}{R_A+R}} = \frac{E(R_A+R)}{(R_A+R)(R+R_L)+R_A R}$$

$$I_2 = I_3 \times \frac{R_A}{R_A+R} = \frac{E\,.\,R_A}{(R_A+R)(R+R_L)+R_A R} \qquad (16)$$

$$\text{Now } I_1 = \frac{E}{2R} \quad .. \quad .. \quad .. \qquad (17)$$

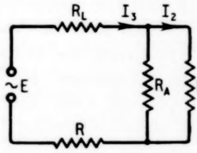
$$\text{and } 20\log_{10}\left|\frac{I_1}{I_2}\right|. = y \text{ (Fig. 3)}$$

$$\text{or } \left|\frac{I_1}{I_2}\right| = \text{antilog}_{10}\left(\frac{y}{20}\right) = F \text{ say.}$$

$$\text{Hence from (16) and (17), } F = \left|\frac{I_1}{I_2}\right| = \frac{(R_A+R)(R_L+R)+R_A R}{R_A R}$$

$$\text{whence } R_A = \frac{R(R_L+R)}{2RF-R_L-2R} = \frac{R_L+R}{2(F-1)-\dfrac{R_L}{R}} \qquad (18)$$

Fig. 7. *Conditions in equalized circuit for zero frequency.*



The advantage of this method is that it does not require a prior determination of w for $f = 0$ and, apart from F, involves only simple resistance values.

Reference

[1] "Design of Attenuation Equalizers," by H. N. Wroe, *Wireless Engineer*, October 1946, Vol. 23, p. 272.

THE TELEVISION SOCIETY

A Midland centre of the Television Society has been formed with headquarters in Birmingham, and details are available from the Lecture Secretary, Dr. W. Summer, 169, Maryvale Road, Bournville, Birmingham 30. Meetings will be held at 7 p.m. on the first Wednesday in every month except during July and August.

PHYSICAL SOCIETY'S EXHIBITION

The travelling-wave tube shown by Standard Telephones & Cables was quoted in the review of the exhibition in the May issue as operating at a frequency of 10,000 Mc/s. This figure should have been 3,000 Mc/s.

LITERATURE

Spontaneous Capacitance Fluctuations in Silvered Ceramic and Silvered Mica Capacitors. By H. F. Church, B.Sc., Technical Report L/T 181. The British Electrical & Allied Industries Research Association, 15, Savoy St., London, W.C.2. Price 6s. (post 3d).

A catalogue describing Douglas and Macadie coil-winding machines has been prepared by The Automatic Coil Winder & Electrical Equipment Co. Ltd., Winder House, Douglas St., London, S.W.1. It gives details of the wide range of winding machines, including paper interleaving models and taping machines, which are produced by this firm.

NEW BOOKS

Über Synchronisierung von Röhrengeneratoren durch Modulierte Signale

By Fritz Diemer. Pp. 98 with 35 illustrations. Gebr. Leemann & Co. Stockerstrasse 64, Zürich 2. Price Fr. 10.80 (Swiss).

This is a doctorate thesis from the Zurich Technical Hochschule. It is well known that a valve oscillator can be held in synchronism by an externally applied voltage if the frequency deviation does not exceed a certain value depending on the oscillator characteristics and the magnitude of the synchronizing voltage. In this thesis the synchronizing voltage is assumed to be either amplitude or frequency modulated. If, for example, the carrier has been partly eliminated at the transmitter, it is necessary to re-insert it at the receiver and the question arises as to the ability of the received modulated signal to hold the locally-generated carrier at the correct frequency, phase, and amplitude.

The first fifty-two pages are devoted to a theoretical investigation and the remainder to the experimental verification of the conclusions. The work is necessarily mathematical and the reader is referred at the beginning to the treatment of Hill's differential equation by Whittaker and Watson and by Strutt in his monograph on Lamé and Mathieu functions. From the more practical point of view special attention is drawn to several papers by Erdélyi in 1934-5.

Tests were made both with amplitude and frequency modulation and the results fully confirmed the theoretical conclusions. Circuit diagrams of the various experiments are given and numerous graphs of the experimental results. This is followed by a discussion of the technical applications of the methods employed, and by a description of an actual 5-W 50-Mc/s relay station employing a synchronized generator with frequency modulation.

The booklet is well produced and constitutes a valuable addition to the literature of this subject.

G. W. O. H.

Microwave Mixers

By R. V. Pound and Eric Durand. pp. 381 + xii, with 221 figures. (Radiation Laboratory Series.) McGraw-Hill Publishing Co., Aldwych House, London, W.C.2. Price 33s. (in U.K.)

This book is Vol. 16 of the M.I.T. Radiation Laboratory Series on radar and related techniques, and it covers most of the developments, chiefly on 10-cm and 3-cm wavelengths, which have evolved during the war years, and up to more recent times. Although this book describes only the American developments, and British developments over the same period can by no means be ignored, nevertheless the book provides comprehensively the theoretical and empirical data upon which mixer design is based, together with descriptions of representative mixers.

Following a preliminary and extensive review of the receiver problem, of crystals, and of the principles of mixer design, the book describes their application to the design of the r.f. head for radar use. Thus, the functioning of waveguide and coaxial-line microwave circuits, including t.r. and anti-t.r. gas switches, and local-oscillator valves and feeds, are discussed and described. The book concludes with a very informative discussion of the automatic-frequency control of microwave receivers, followed by a brief chapter on mixer measurements.

There is, spread throughout the book, a very good theoretical and empirical analysis of the effects upon the noise factor of a receiver of variations in the design features, and also an excellent treatment of the significance in relation to mixer design of r.f. and i.f. admittances. For example, the effect of the local oscillator and of different forms of local-oscillator coupling in relation to receiver noise factor, and also of the relative importance of image-frequency terminations, are both thoroughy discussed. Multiple mixers for a.f.c., beacon, and signal circuits are described; and an extensive description of the Magic-T and its applications in receiver design is also given.

It is refreshing, in a text of this kind, to find waveguide and coaxial-line diagrams in which exact dimensions and mechanical tolerances are given. This book contains an appreciable number, and one could wish that even more had been given.

It is unfortunate that this book, containing such excellent subject matter, is not equally well written. Because of indiscriminate and lengthy mixing of one subject matter with another, and of the lesser with the more relevant details, it is difficult to sort out any detailed information which the reader may require and, occasionally, to see what the author is driving at. There is also much repetition of statements of fact and of descriptions of design principles. Indeed, in some cases, almost identical sentences are repeated on different pages.

The experienced worker who can sort out the book for himself will find it very useful for reference. The beginner will find it necessary to study the book completely, and not in stages, in order to obtain an understanding of the salient features of the subject.

L. W. B.

Television Explained

By W. E. Miller. Pp. 50 with 56 illustrations. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 3s. 6d.

The various parts of a television set are simply explained and the book provides an introduction to television which is suited to those acquainted with ordinary wireless equipment. Covering the receiver stage-by-stage, it deals with aerials, r.f., i.f. and v.f. amplification, sync separation, time bases for electrostatic and electromagnetic tubes, cathode-ray tubes and power supplies.

Radar Aids to Navigation

Edited by John S. Hall. Pp. 389+xiii with 192 illustrations. (Vol. 2, Radiation Laboratory Series.) McGraw Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price 30s. (in U.K.)

Fundamentals of Radar

By Stephen A. Knight, F.R.S.A. Pp. 128+vii with 97 illustrations. Sir Isaac Pitman & Sons Ltd., Kingsway, London, W.C.2. Price 10s.

Introduction to Wireless

By W. F. Pearce, B.Sc. Pp. 247+viii with 175 illustrations. G. Bell & Sons Ltd., York House, Portugal St., London, W.C.2. Price 7s. 6d.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Standard Terms and Abbreviations

SIR,—Your Editorial of March 1948 on " Standard Terms " and, in particular, your reference to the faulty definition of magnetic susceptibility in the 1936 edition of the B.S.I. glossary, prompts me to point out that a serious error remains uncorrected in the 1943 edition where mass susceptibility (Definition 1259) is stated to be " the quotient of the susceptibility and the magnetic flux density." Presumably the printer, seeing the word 'density' in the definition, thought that it must mean magnetic flux density, and altered it accordingly.

Another standard term which merits consideration is ' magnetise ' (Definition 1254). It is the present practice of this firm to spell it with the -ise ending, following the authority of the B.S.I. glossary, and preserving the similarity to ' magnetism.'

Should it be -ise or -ize ? The suffix is derived from the Greek -izo, therefore the Oxford English Dictionary gives ' magnetize', reserving -ise for verbs like ' advertise' and ' surprise ' which do not contain the Greek suffix. Fowler's " Modern English Usage," the " Encyclopaedia Britannica," and most printing houses including *The Times*, also insist on ' magnetize,' which is universal in the United States, so often the source of spelling innovations.

Treble and Vallin's " A.B.C of English Usage " does 'advise' 'magnetise,' thinking it better to ' standardise' on one spelling than to insist on a pedantic division between those verbs which are entitled to -ize and those which are not. This may have influenced the compilers of the Glossary, but if I write ' magnetise' in an article for publication, the printer would probably correct the spelling in spite of the B.S.I. definition. Is it desirable that this should continue ? Which is the higher authority, B.S.I. or O.E.D. ?

ALAN EDWARDS

James Neill & Co. (Sheffield) Ltd.,
Sheffield.

SIR,—As the author of " Absolute Bels " published in your Journal (May, 1946) I must protest against Mr. Odell's letter in your May issue.

As the absolute decibel gives more information than the decibel, which is merely a power ratio, I purposely chose " dB " for the former in order to distinguish this clearly from the accepted nomenclature of B.S.661. The use of the capital letter for decibel at once removes this useful distinction.

F. S. G. SCOTT.

Rugby, Warwickshire.

A Matter of Terminology

SIR,—The publication, in your April issue, of two papers dealing with " polar diagrams " prompts me to draw your attention to the somewhat loose way in which this term is used by writers, and the consequent danger of confusion.

I understand a polar diagram to be one in which a point is located by the use of polar co-ordinates—a radius vector and an angle. In the case of the " Polar Diagram of an Aerial," I understand that the *angle* is a direction from the aerial, but the nature of the *radius vector* is in doubt—it may represent field strength, power, or some function of either. Here is one source of confusion.

When, in addition, I read the caption " polar diagram " under a diagram drawn in Cartesian co-ordinates, I am still more puzzled.

Some examples drawn from the above-mentioned papers :—

Page 103. " The horizontal polar diagram will be circular."

This remark, taken in conjunction with Fig. 2 (where Cartesian co-ordinates are used for a " polar diagram ") suggests the question " What does a circle look like when drawn in Cartesians ? "

Page 114. " . . . a Gaussian function . . . for the polar diagram of a directive aerial. Fig. 7 shows such a function."

Fig. 7 is drawn in Cartesians.

The author of the second article is properly careful as regards the captions of his figures : Cartesian curves are called " radiation patterns " ; polar co-ordinate curves are called " polar diagrams."

An Interservice committee, which started meeting during the war, realized this difficulty, and inserted the following definition in a Radio Glossary, published for " Service " use in October 1946.

Polar diagram. A mathematical term for a diagram using *polar co-ordinates*. Its colloquial *unqualified* use is deprecated.

May I suggest the use of the following terms :—

Radiation (etc) pattern (in any co-ordinate system) (qualified, in addition by " horizontal," etc., if necessary).

The term " polar " to be added (if necessary) if polar co-ordinates are used.

Haslemere, L. H. BAINBRIDGE-BELL.
Surrey.

Aerial Gain and Polar Diagrams

SIR,—I have read with interest the paper " Determination of Aerial Gain from its Polar Diagram " by J. A. Saxton, published in the April issue of your journal.

In 1945 a paper on the same subject was prepared by S. Martin and myself as an internal report at S.R.D.E. In this paper we have given formulae for calculating the gain of a highly-directive aerial system derived from a knowledge of the half-power or half-voltage beam width of the aerial ; i.e., in a form corresponding to equation (31) of Dr. Saxton's paper.

Dr. Saxton has considered an ellipsoid of revolution as the approximation to the actual polar diagram of a highly-directive aerial system. In our paper we have discussed two slightly different approximations (a) a cone terminated by one-half of an ellipsoid and (b) an elongated spheroid with an envelope of the form $E = E_0 \cos n\theta$.

It was found that the " elongated spheroid " results were in very good agreement with experimental data obtained for paraboloidal reflectors whereas the "cone-ellipsoid" approximation gave too high gains. The gains calculated from Saxton's formulae appear to be slightly smaller than those obtained by us. In our paper calculations were also given of the gain of a paraboloidal reflector fed by a radiating waveguide as compared with dipole feed.

We hope to publish these results shortly.

B. J. Starnecki

Signals Research and Development
Establishment, Christchurch, Hants.

Intensity-Distance Law of Radiation

Sir,—In radio work, even with centimetre waves and 'optical' mirrors, it is normally assumed that the performance of the aerial system can be expressed in the usual way as a gain of so many db over an omni-directional aerial. This implies that the field-strength versus distance law is the same for the beam as for omni-directional radiation (otherwise the gain of the directional aerial would appear to vary at different ranges), and therefore the energy intensity in a beam radiated into free space should vary inversely as the square of the distance from the source. But on the other hand it has been pointed out that the beam from an optical searchlight maintains its intensity at a much higher intensity than would be indicated by the inverse square law. The question then arises, what is the dividing line between the optical case and the radio case?

If one had a perfectly parallel beam (to which the optical searchlight is an approximation) the cross-section of the beam would be the same at all distances and *equal to the aperture area of the source*, and the intensity would then be reduced only by absorption in the medium of propagation. But radio beams normally have a beam-width which can be specified as an *angle*; i.e., the volume occupied by the beam approximates to a cone rather than a cylinder. In the cone, the area of section increases as the square of the distance, and the energy intensity in such a beam will obey the inverse square law. Now the limit to the conical approximation to beam shape is that the finite size of the source prevents the cone from extending back to its apex, and the condition for it to apply is therefore that the distance from the source should be sufficient to make the beam section large compared with the source aperture. The beam diameter is equal to $r\theta$ where θ is the angle of divergence and r the distance from the vertical source at the apex of the cone, so the condition for the inverse square law to apply is $r\theta \gg d$ where d is the diameter of the aperture of the actual source. But θ is a function of ratio of wavelength to aperture diameter, and if θ is defined as the half-power point, $\theta = \sin^{-1}(0.52\lambda/d) \approx 0.52\lambda/d$ (if $\lambda \ll d$) for ideal illumination of the mirror. The required condition is therefore $0.52r\lambda/d \gg d$ or $0.52r\lambda/d^2 \gg 1$. An appropriate 'critical distance' parameter might then be $r_0 = d^2/0.52\lambda$.

Perhaps the sharpest radio beam that might be considered with present technique corresponds to $\lambda = 0.01$ metre (1 cm) and $d = 2$ metres. Then $r_0 \approx 800$ metres. If λ were increased to 0.1 metre (10 cm) with the same mirror diameter, r_0 would be 80 metres. With an optical searchlight of 1-metre diameter, and $\lambda \approx 5 \times 10^{-7}$ metre, $r_0 \approx 4 \times 10$ metres. It is evident that in the optical system the limit is set by imperfections in the apparatus (e.g., finite size of light source) while in most v.h.f. radio systems the diffraction effect prevents the achievement of a truly parallel beam and introduces inverse square law attenuation at most practical working distances.

D. A. Bell.

British Telecommunications Research Ltd.,
Taplow.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

589 834.—Phonograph reproducer in which the movements of the stylus are applied to detune separate high-frequency circuits, which are coupled to a pair of balanced diode rectifiers.

Radio Corporation of America. Convention date (U.S.A.) 29th March, 1944.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

589 809.—Directional receiver, for use with a beam transmitter, in which a rotary aerial system feeds a cathode-ray indicator calibrated by marker signals.

Hazeltine Corporation (assignees of B. D. Loughlin). Convention date (U.S.A.) 20th September, 1943.

589 958.—Toroidal resonator which is periodically impulsed to control the flow of energy through one or other of the limbs of a forked waveguide feeding the aerial of a radiolocation set.

J. D. Cockcroft and J. Ashmead. Application date 27th August, 1943.

590 065.—Directional receiver in which the positional indications shown are automatically repeated at a remote point.

Hazeltine Corporation (assignees of B. D. Loughlin and J. F. Craib). Convention date (U.S.A.) 20th September, 1943.

590 206.—Device to maintain sufficient water-vapour pressure in the gas-discharge tube used for switching the aerial feeders in radiolocation sets.

G. B. Banks and J. Buckingham. Application date 11th January, 1945.

590 260.—Automatic direction-finder in which the cathode-ray indication is shown as a radial line-trace, this being produced by periodically switching the steady voltages normally applied to the deflecting plates of the tube.

Standard Telephones and Cables Ltd. and R. F. Cleaver. Application date 14th April, 1945.

590 261.—Means for repeating at a remote point cathode-ray indications of the kind described in the preceding patent (No. 590 260).

Standard Telephones and Cables Ltd. and R. F. Cleaver. Application date 14th April, 1945.

590 413.—Directive aerial of the single-slot type with coupling means designed to ensure efficient operation over a wide frequency-band.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 1st January, 1945.

590 459.—Marking-out a navigational course, particularly for surface vehicles, by radiating pulses in predetermined time-relation from beacon stations located on opposite sides of the course.

Standard Telephones and Cables Ltd. (assignees of A. Alford). Convention date (U.S.A.) 8th October, 1942.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

589 898.—Diversity receiving system in which the outputs from two spaced aerials are so coupled to a common load as to give substantially the benefit of a third aerial.

Marconi's W.T. Co. Ltd. (assignees of W. I. Matthews). Convention date (U.S.A.) 30th March, 1944.

589 968.—Automatic gain-control system, particularly for receiving interrupted c.w. signals, wherein the optimum level is determined by each signal-impulse.

Marconi's W.T. Co. Ltd. (assignees of R. L. Hollingsworth). Convention date (U.S.A.) 30th December, 1943.

590 203.—Impedance-transforming network, comprising two coils wound partly in parallel and partly in series, for matching say a push-pull amplifier to an aerial.

" Patelhold " Patentverwertungs etc. A.G. Convention date (Switzerland) 16th October, 1943

590 209.—Diversity receiving system with means for automatically selecting the best signal combination, whilst preventing signal cancellation or distortion.

Marconi's W. T. Co., Ltd., (assignees of M. G. Crosby). Convention date (U.S.A.) 14th January, 1944.

590 210.—Frequency-discriminating circuit, comprising a pair of grid-controlled rectifiers and means for preventing degenerative feedback, for receiving frequency-modulated signals. (addition to 561954).

Marconi's W. T. Co., Ltd., K. R. Sturley and B. Easter. Application date 24th January, 1945.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

590 039.—Process for depositing an insulating film on the mosaic screen of a television storage camera.

Cinema-Television Ltd., and R. B. Head. Application date 10th April, 1945.

590 215.—Transitron valve, with time-delay network, for generating equally-spaced pulses, suitable for synchronizing, say in television.

E. W. Anderson and T. J. McDermott. Application date 16th February, 1945.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

590 066.—Multi-channel signalling system utilizing pulse-phase modulation, with a distinctive series of pulsed signals for synchronizing the transmitter and receiver.

D. G. Reid. Application date 15th August, 1944.

590 067.—Multi-channel signalling system, utilizing centimetre waves and sequences of pulse-groups, in which fixed intervals occur between individua pulses with variable intervals between the groups.

D. G. Reid and P. J. Nowacki. Application date 15th August, 1944.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

589 580.—Process for making a metal-to-metal seal in the manufacture of electron-discharge tubes.

Standard Telephones and Cables Ltd., and C. N. Smyth. Application date 14th March, 1945.

SUBSIDIARY APPARATUS AND MATERIALS

589 592.—Composition and preparation of crystal rectifiers and mixers of the silicon type.

The General Electric Co., Ltd., C. E. Ransley, J. W. Ryde and S. V. Williams. Application dates 28th May, 18th July and 20th August, 1941.

589 606.—Delay network comprising polarity-reversing means for triggering a flip-flop multivibrator in the process of re-conditioning a train of pulses of irregular shape and amplitude.

D. G. Reid and C. C. Cradwick. Application date 20th July, 1944.

589 787.—Variable tuning inductance of the slide-wire type particularly suitable for coils of small diameter.

The Plessey Co., Ltd. Convention date (U.S.A. 5th May, 1944.

589 935.—Shaped and perforated strip-elements adapted to be conveniently inter-engaged, so as to form a support or chassis for wireless components.

A. M. Jones. Application date 3rd April, 1945.

589 952.—Stack of metal-contact rectifiers so spaced and supported as to provide adequate cooling space between adjacent electrodes.

Standard Telephones and Cables Ltd., (assignees of A. J. Miller). Convention date (U.S.A.) 24th April, 1944.

590 242.—Means for measuring the width or duration of rectangular pulses by applying them to shock-excite a tuned circuit.

Standard Telephones and Cables Ltd., (assignees of E. Labin and D. D. Grieg). Convention date (U.S.A.) 26th March, 1943.

590 278.—Application of the " contrast-inversion " scanning process, as used in television, to project a positive picture from a cinematographic negative, directly after development.

Cinema Television Ltd., and A. G. D. West. Application date 18th October, 1944.